Atmospheric Predictability: From Basic Theory to Forecasting Practice.

Eugenia Kalnay

Alghero, May 2008, Lecture 1

We honor Ed Lorenz (1917-2008) who started the whole new science of predictability

NATURE|Vol 453|15 May 2008

OBITUARY

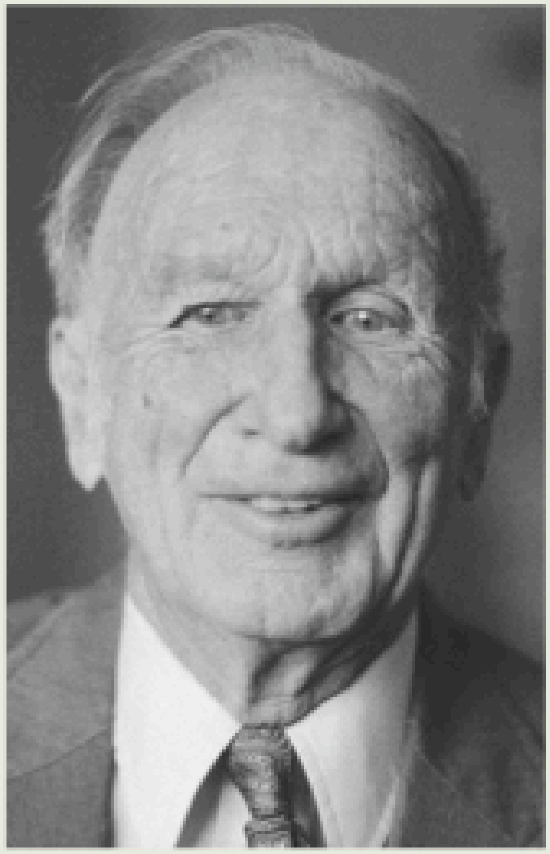
Edward N. Lorenz (1917–2008)

Meteorologist and father of chaos theory.

Edward Norton Lorenz, whose pioneering studies of atmospheric dynamics led to his accidental discovery of chaos theory, died of cancer at his home in Cambridge, Massachusetts, on 16 April. A modest, unassuming and kind man, his personal qualities and intellectual insights had been a constant feature in the field of meteorology for more than 60 years; he co-authored his last paper just weeks before his death.

Born on 23 May 1917 in West Hartford, Connecticut, Lorenz took bachelor's and master's degrees in mathematics at Dartmouth College, New Hampshire, and Harvard University, respectively. Service as a weather forecaster for the US Army Air Corps during the Second World War led him into meteorology, and he received a doctorate in the subject at the Massachusetts Institute of Technology (MIT) in 1948. He remained in MIT's Department of Meteorology for the rest of his academic career, becoming emeritus professor there in 1987.

Lorenz made crucial contributions to atmospheric science, many of which are still routinely taught to students and widely used in weather forecasting. Perhaps foremost among these is his formulation in



at a consistent rate. When Lorenz plotted variables representing temperature and flow against one another, the system eventually adopted trajectories that traced out something akin to a pair of

nonlinear systems such as the atmosphere. Most immediately for Lorenz's field, this meant that long-term weather predictions were impossible, because the atmosphere's initial state can never be specified precisely enough. That was a situation that increased computing power could not change.

Lorenz perfectly encapsulated this unknowability in the title of a talk that he gave to the American Association for the Advancement of Science in 1972. The question it asks has since lodged itself in the public's consciousness: "Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?" But the influence of chaos theory extends far beyond meteorology, and much deeper: it challenges the entire deterministic world view, as was confidently expressed, for instance, by the mathematician and philosopher Pierre-Simon Laplace, who stated at the beginning of the nineteenth century that the entire future could be determined by constructing and solving the equations governing all components of the Universe.

Although the existence of chaos had been recognized before Lorenz — notably in the 1890s by Henri Poincaré, in his study of the motions of three or more gravitating celestial

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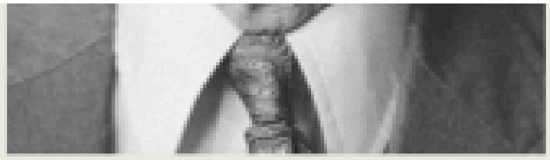
Lorenz made crucial contributions to atmospheric science, many of which are still routinely taught to students and widely used in weather forecasting. Perhaps foremost among these is his formulation in the mid-1950s of the concept of 'available potential energy', which he used to explain how potential energy and kinetic energy are interchanged in the atmosphere. His application of these ideas culminated in his influential book of 1967, *The Nature and Theory of the General Circulation of the Atmosphere*. He was also instrumental in the development of numerical techniques for weather prediction. One example — again, still widely used — is his scheme for the numerical treatment of changes in atmospheric variables with height, now known as the Lorenz vertical grid.

But the work for which Lorenz is undoubtedly most widely known is a now-classic paper published in the *Journal of Atmospheric Science* in 1963. Entitled 'Deterministic nonperiodic flow', it presented surprising results from a simplified computational model that simulated thermal convection in a fluid layer heated from below and cooled from the top. The calculated flow of the fluid was extremely irregular, with almost random qualities. But more importantly, it exhibited extremely sensitive dependence on initial conditions: two fluid states that were at first just slightly different diverged from each other exponentially, with their differences doubling repeatedly



at a consistent rate. When Lorenz plotted variables representing temperature and flow against one another, the system eventually adopted trajectories that traced out something akin to a pair of butterfly wings — a pattern since called the Lorenz attractor. He further observed that the system trajectory moved from one wing of the butterfly to another in a seemingly erratic manner.

In his book *The Essence of Chaos*, Lorenz recounts how he came to discover the extreme sensitivity of his model to small changes. Wishing to repeat his simulation, he restarted it with numbers that had been printed out for the start conditions, and left it to go down the hall to fetch a cup of coffee. On his return, he found that the result was nothing like the previous one. He soon identified the reason: the numbers from the print-out were rounded off. In the course of a coffee break, that small error had propagated with exponential speed to change the result completely.

This discovery was epoch-making for two reasons. The first lay in Lorenz's integration of analytical methods with computational simulations, with which he — albeit with a pre-1960 computer that was bulkier, noisier and vastly slower than the PCs of today — set an early precedent for a mode of research that has since become a norm. But much more profound ramifications stemmed from Lorenz's realization of just how general the types of motion he had uncovered were in

philosopher Pierre-Simon Laplace, who stated at the beginning of the nineteenth century that the entire future could be determined by constructing and solving the equations governing all components of the Universe.

Although the existence of chaos had been recognized before Lorenz — notably in the 1890s by Henri Poincaré, in his study of the motions of three or more gravitating celestial bodies — it was Lorenz's meteorological demonstration and analysis that established the universal applicability of the concept, and earned him the title 'the father of chaos'. But it took a decade for chaos theory to percolate through to the general scientific community. When it finally did, it launched a revolution, rapidly extending its sway into many fields of physics, chemistry, biology and engineering — and, in doing so, becoming part of the popular lexicon.

Lorenz received many honours and prizes in recognition of his work, among them the Crafoord Prize — established by the Royal Swedish Academy of Sciences to recognize work in fields not covered by the Nobel prizes — in 1983, and the Kyoto Prize in 1991. The citation for that prize lauded "his boldest scientific achievement in discovering 'deterministic chaos', a principle that has profoundly influenced a wide range of basic sciences and brought about one of the most dramatic changes in mankind's view of nature since Sir Isaac Newton".

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ca. 1974

“Well, we cannot live forever…”

(Lorenz to a friend, 2008)

Chaos in Numerical Weather Prediction and how we fight it

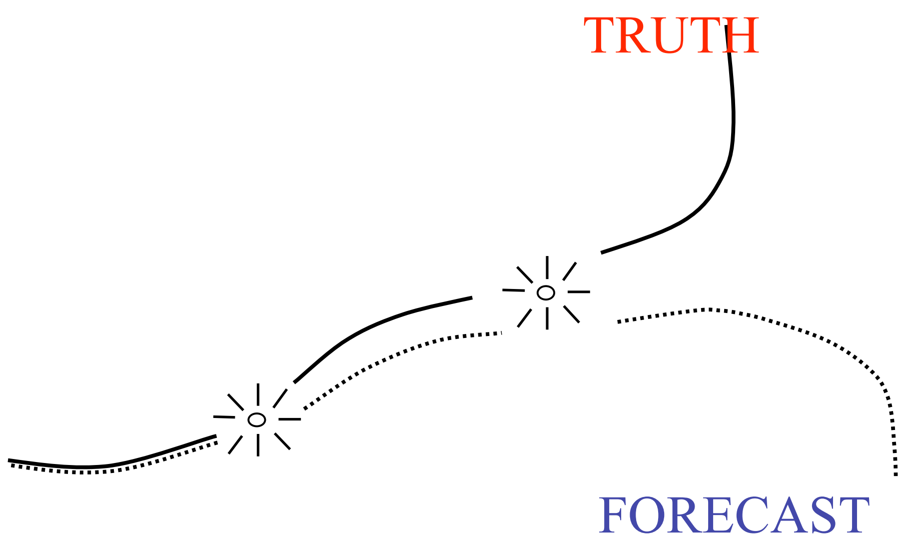
- Lorenz (1963) introduced the concept of “chaos” in meteorology. (Yorke, 1975, coined the name chaos)
  - Even with a perfect model and perfect initial conditions we cannot forecast beyond two weeks: butterfly effect
  - In 1963 this was only of academic interest: forecasts were useless beyond a day or two anyway!
  - Now we exploit “chaos” with ensemble forecasts and routinely produce skillful forecasts beyond a week
  - The El Niño coupled ocean-atmosphere instabilities are allowing one-year forecasts of climate anomalies
- “Breeding” is a simple method to explore and fight chaos
  - Undergraduate interns found that with breeding they could easily predict Lorenz regime changes and their duration
- Chaos-Weather research led to the UMD Local Ensemble Transform Kalman Filter (LETKF, Hunt et al., 2007)

Central theorem of chaos (Lorenz, 1960s):

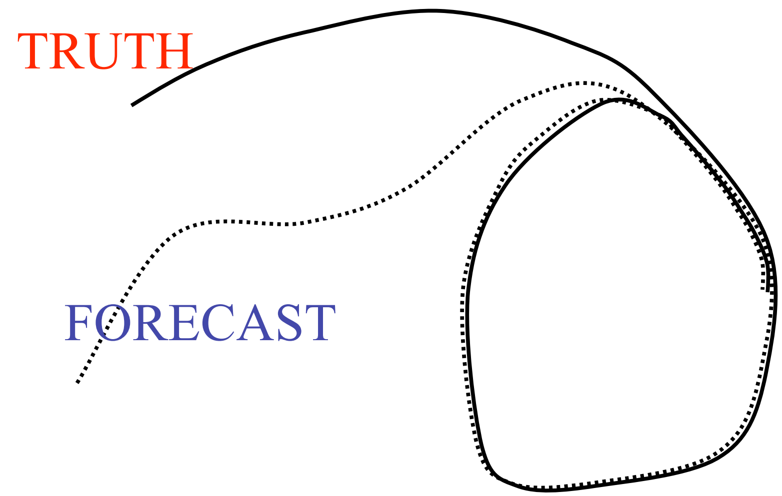
a) Unstable systems have finite predictability (chaos)

b) Stable systems are infinitely predictable

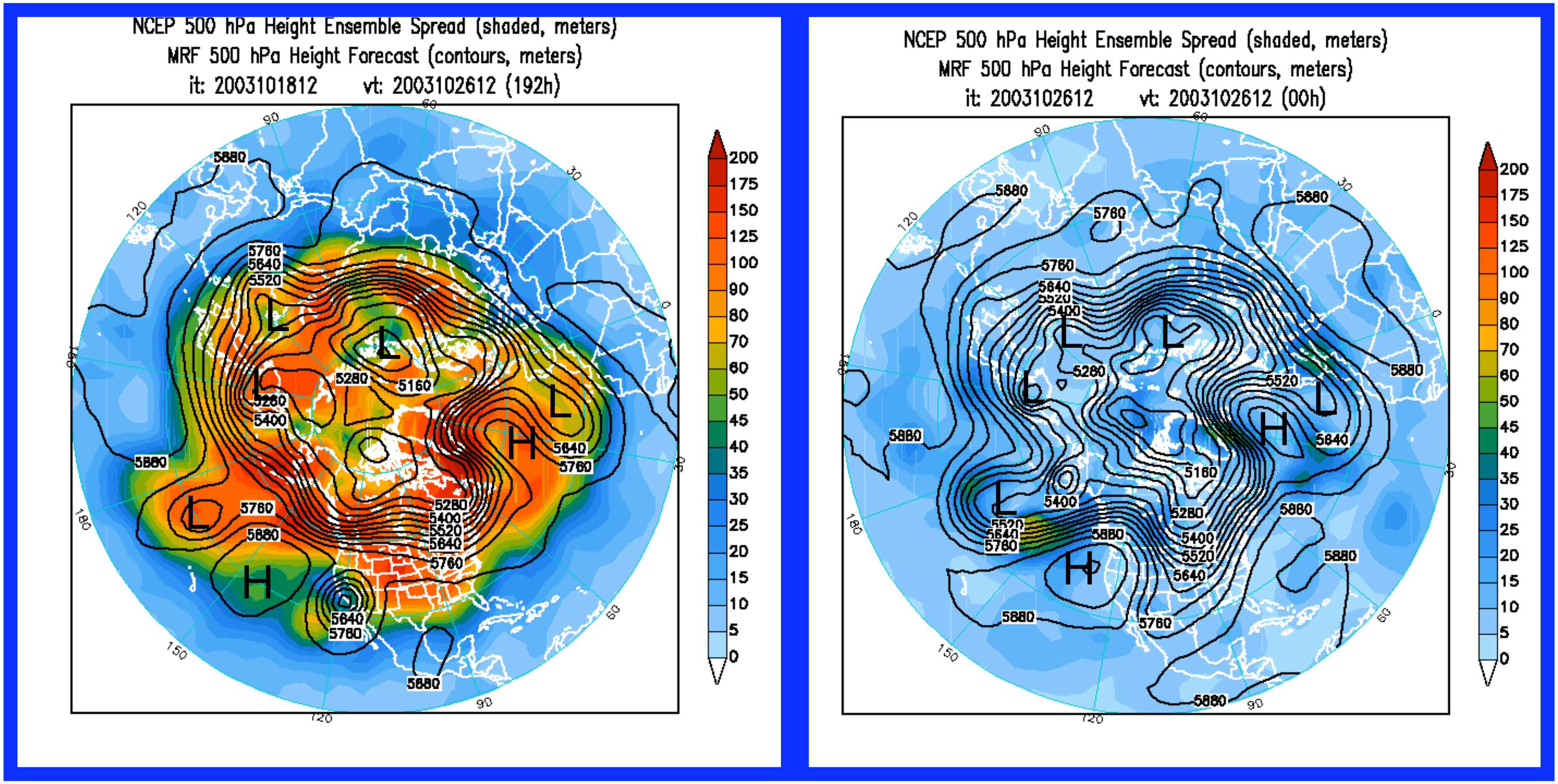
a) Unstable dynamical system



b) Stable dynamical system



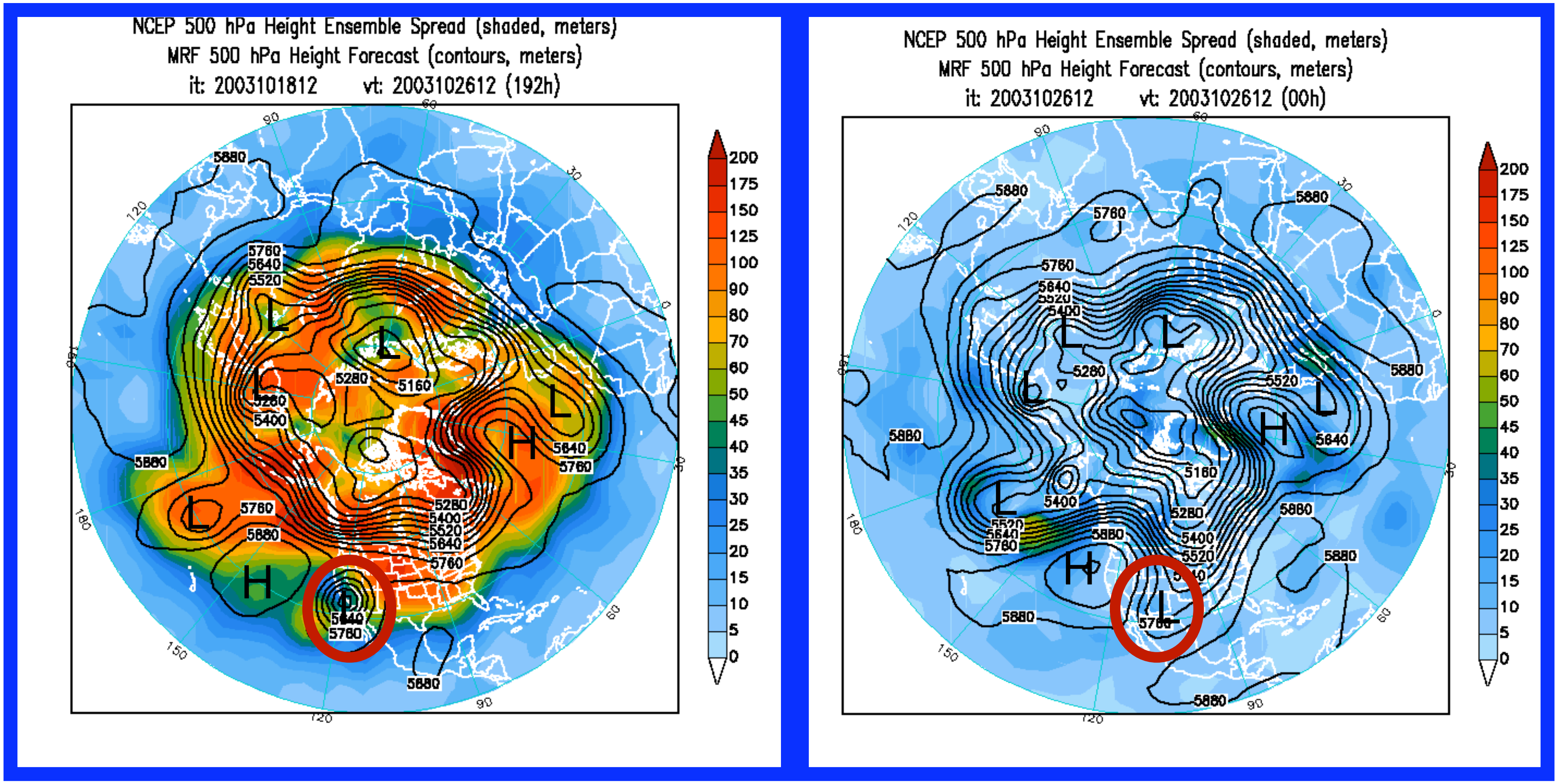
8-day forecast and verification



Almost all the centers of low and high pressure are very well predicted after 8 days!

Need good models, good observations, good data assimilation

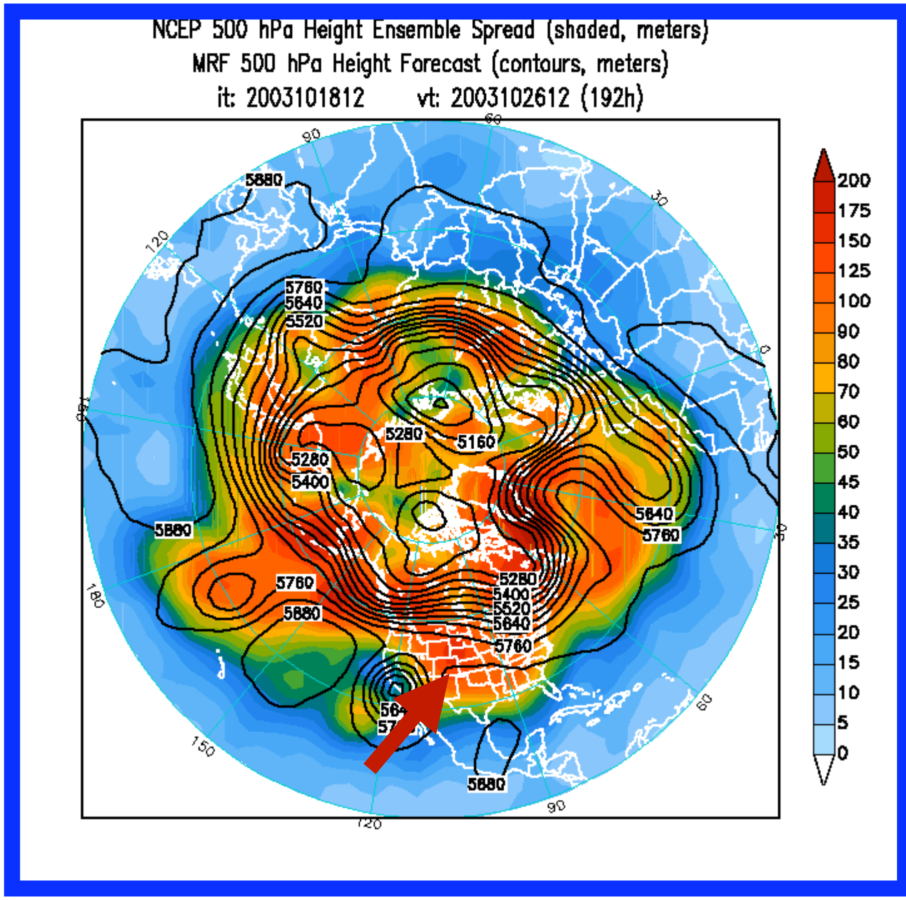
8-day forecast and verification

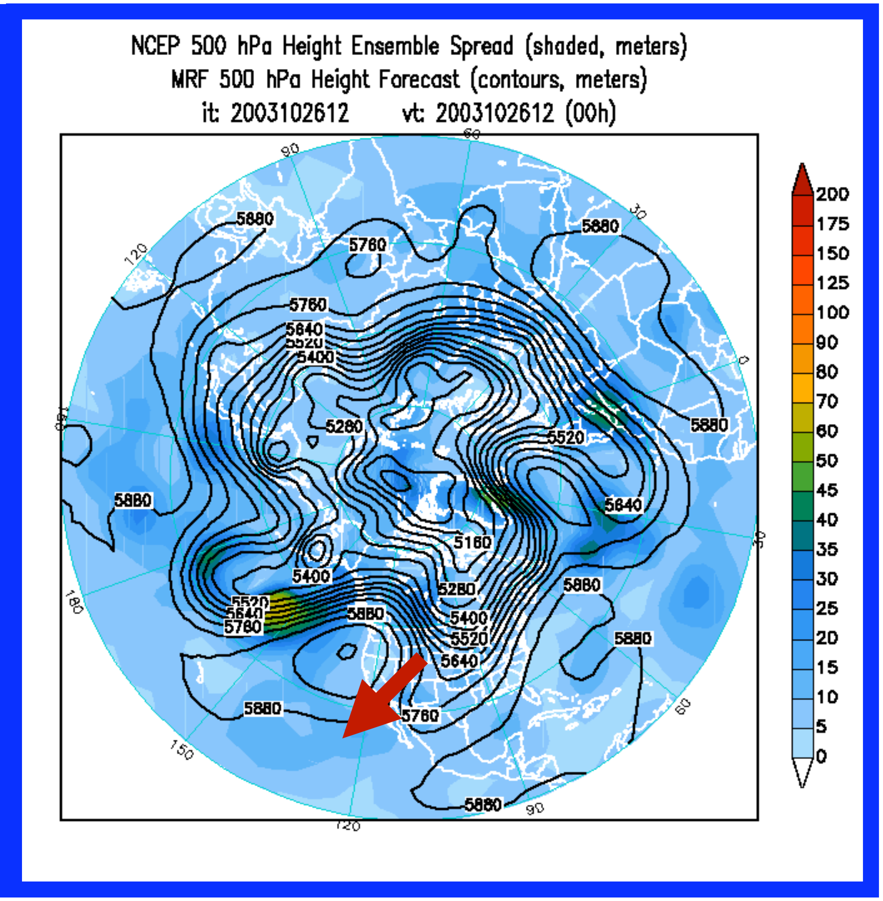


Almost all the centers of low and high pressure are very well predicted after 8 days!

Over Southern California forecast has a cut-off low, not a trough

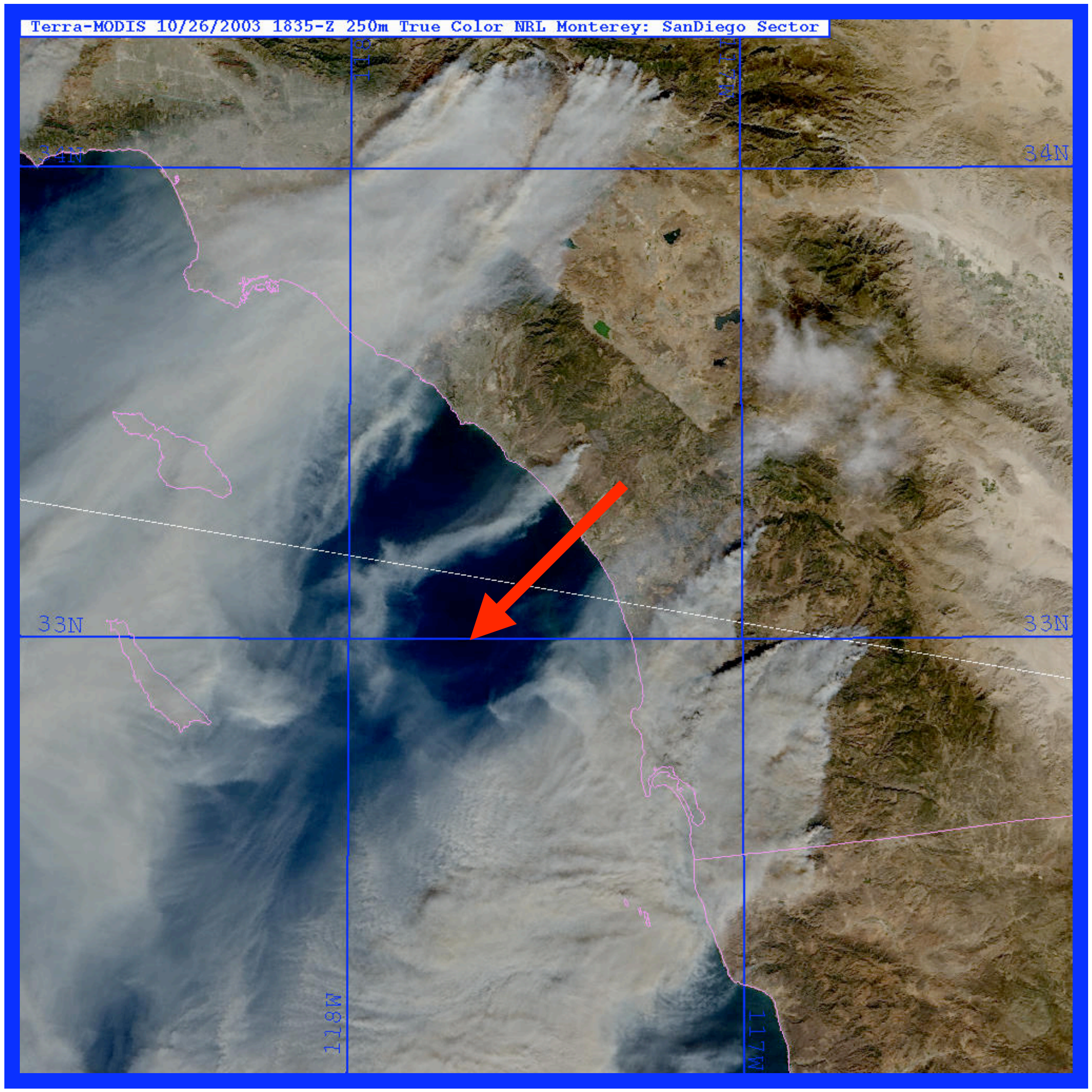
8-day forecast and verification





Southern California: **winds** are from the wrong direction!

Fires in California (2003)



Santa Ana winds: locally wrong prediction (8 days in advance!)

Deterministic Chaos (what!?)

In 1951 Charney indicated that forecast skill would break down, but he attributed it to model errors and errors in the initial conditions…

In the 1960's the forecasts were skillful for only one day or so.

Statistical prediction was equal or better than dynamical predictions,

Like it has been until now for ENSO predictions!

Lorenz wanted to show that statistical prediction could not match prediction with a nonlinear model for the Tokyo (1960) NWP conference

So, he tried to find a model that was not periodic (otherwise stats would win!)

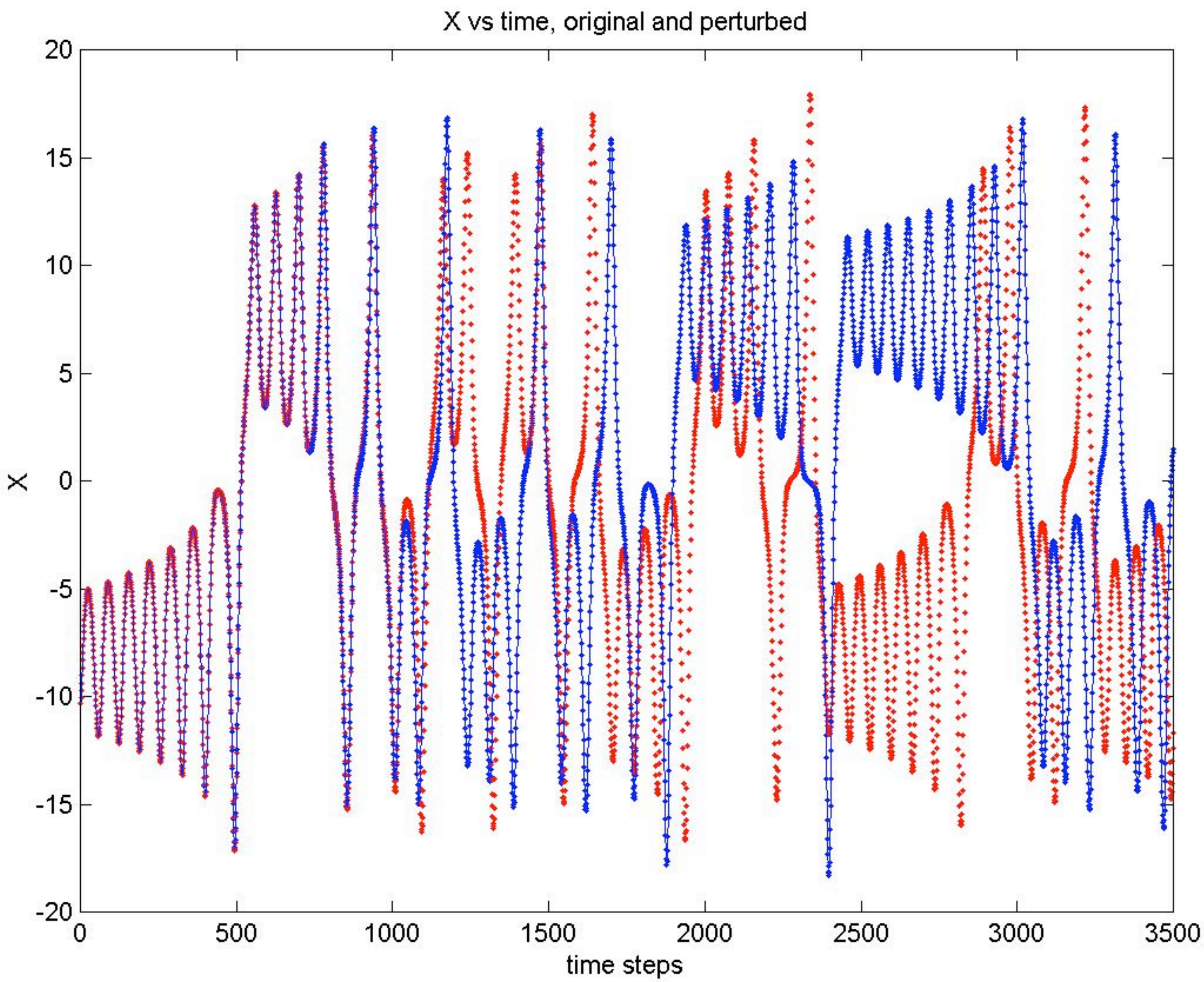
He programmed in machine language on a 4K memory, 60 ops/sec Royal McBee computer

He developed a low-order model (12 d.o.f) and changed the parameters and eventually found a nonperiodic solution

Printed results with 3 significant digits (plenty!)

Tried to reproduce results, went for a coffee and OOPS!

Lorenz introduced an infinitesimal perturbation in the initial conditions, and the two solutions diverged



Lorenz (1963) discovered that even with a perfect model and almost perfect initial conditions the forecast loses all skill in a finite time interval: "A butterfly in Brazil can change the forecast in Texas after one or two weeks".

In the 1960's this was only of academic interest: forecasts were useless in two days

Now, we are getting closer to the **2 week limit of predictability**, and we have to extract the maximum information

<u>Central theorem of chaos (Lorenz, 1960s):</u>

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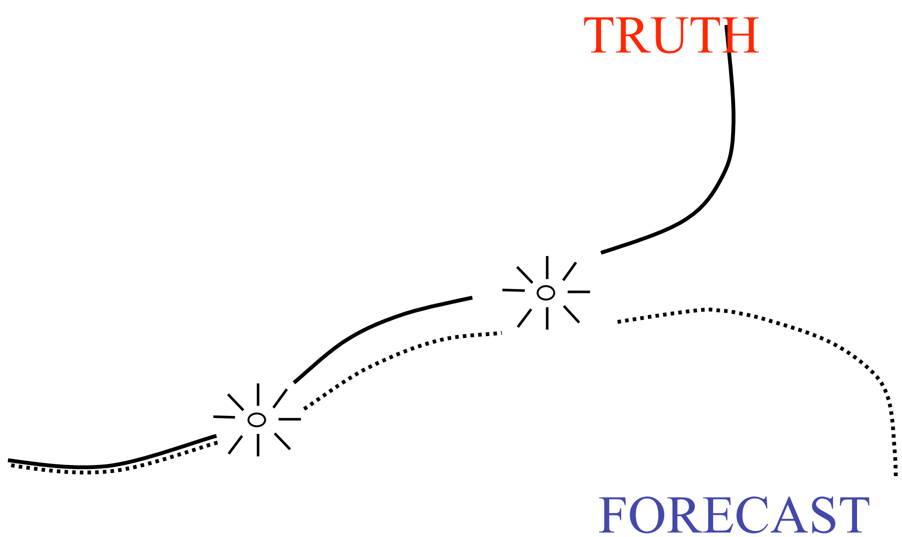
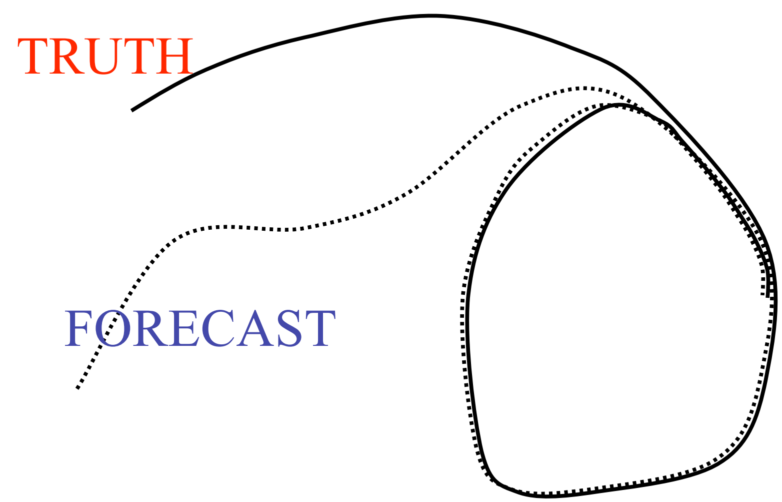


b) Stable dynamical system



A simple chaotic model: Lorenz (1963) 3-variable model

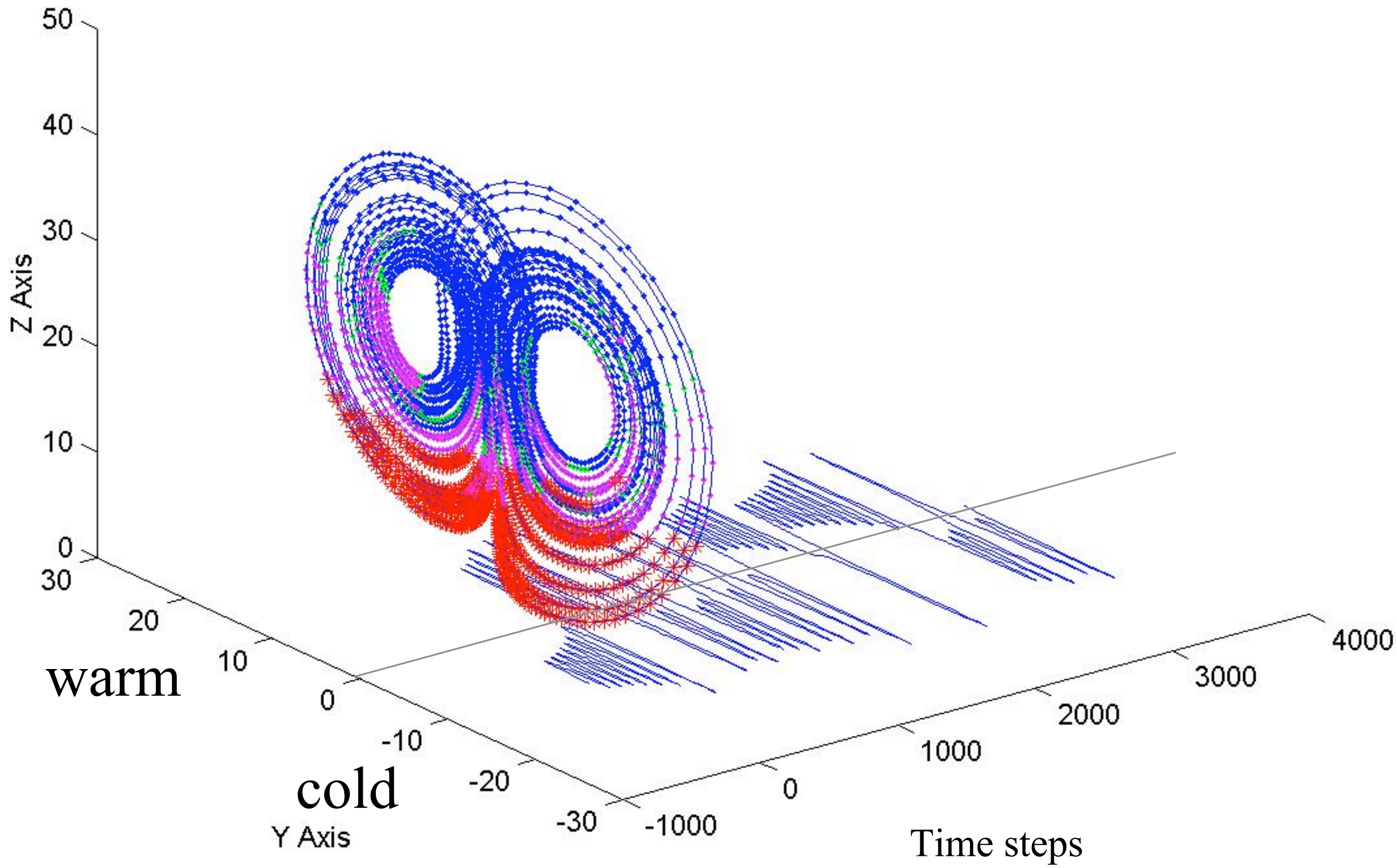
Has two regimes and the transition between them is chaotic

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$

Example: Lorenz (1963) model, y(t)



Definition of Deterministic Chaos

(Lorenz, March 2006, 89 yrs)

WHEN THE PRESENT DETERMINES

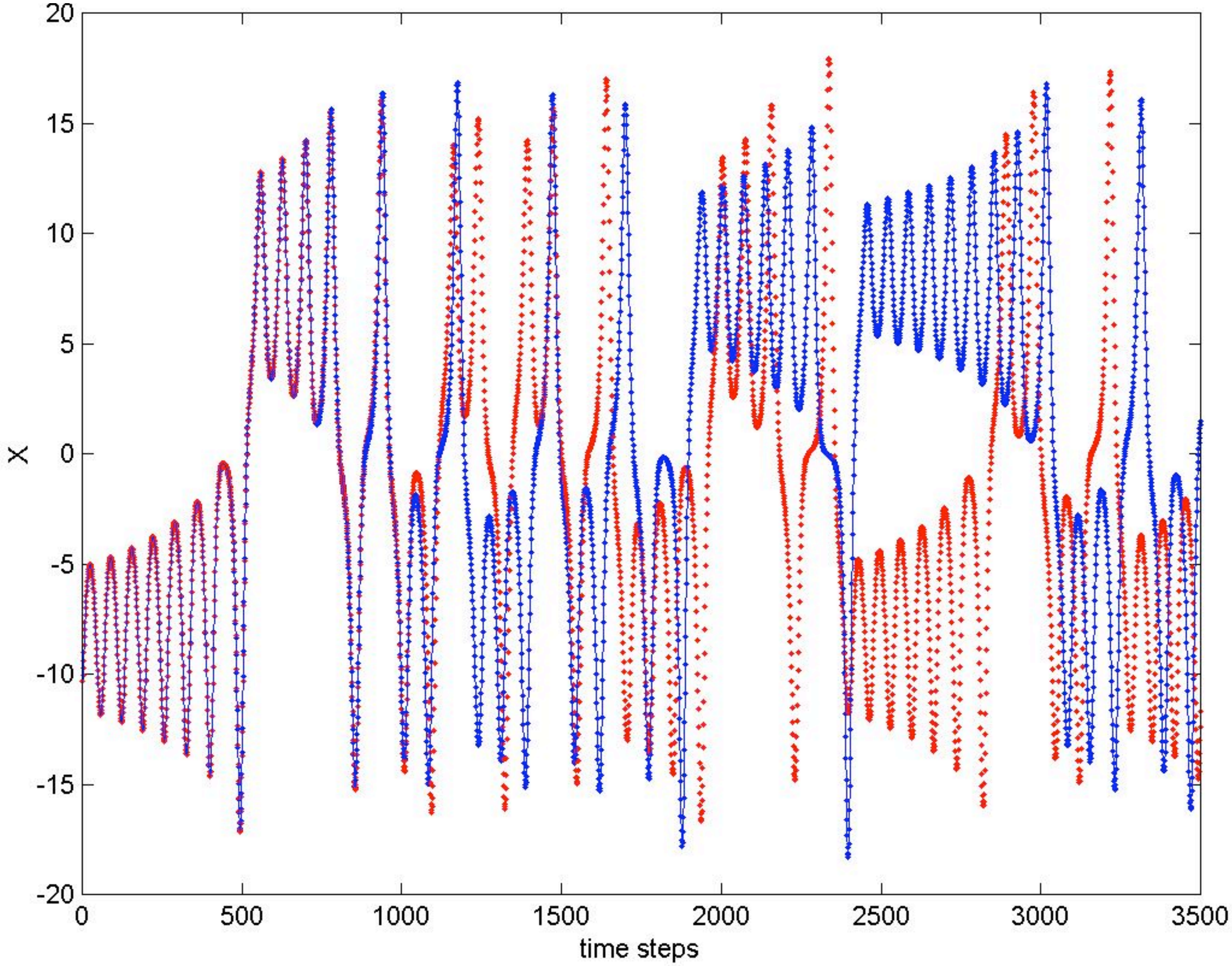
THE FUTURE

BUT

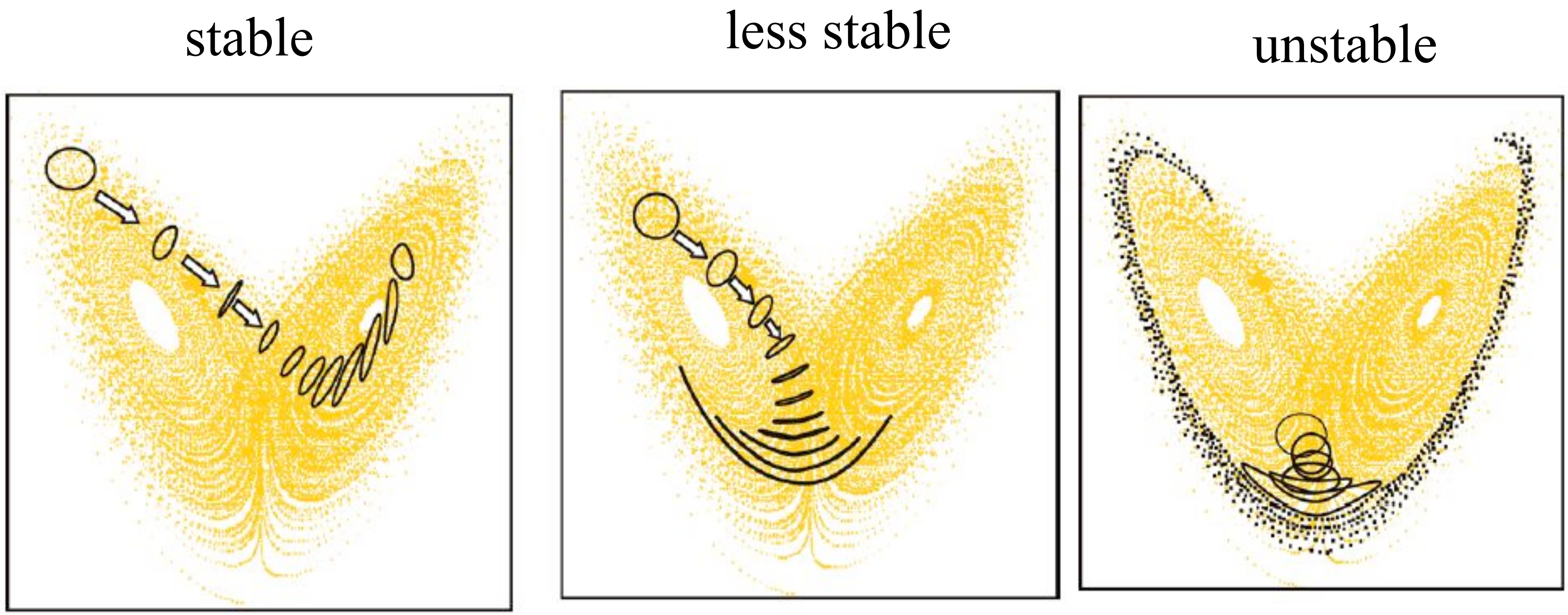
THE APPROXIMATE PRESENT DOES NOT

APPROXIMATELY DETERMINE THE FUTURE

We introduce an infinitesimal perturbation in the initial conditions and soon the forecast loses all skill

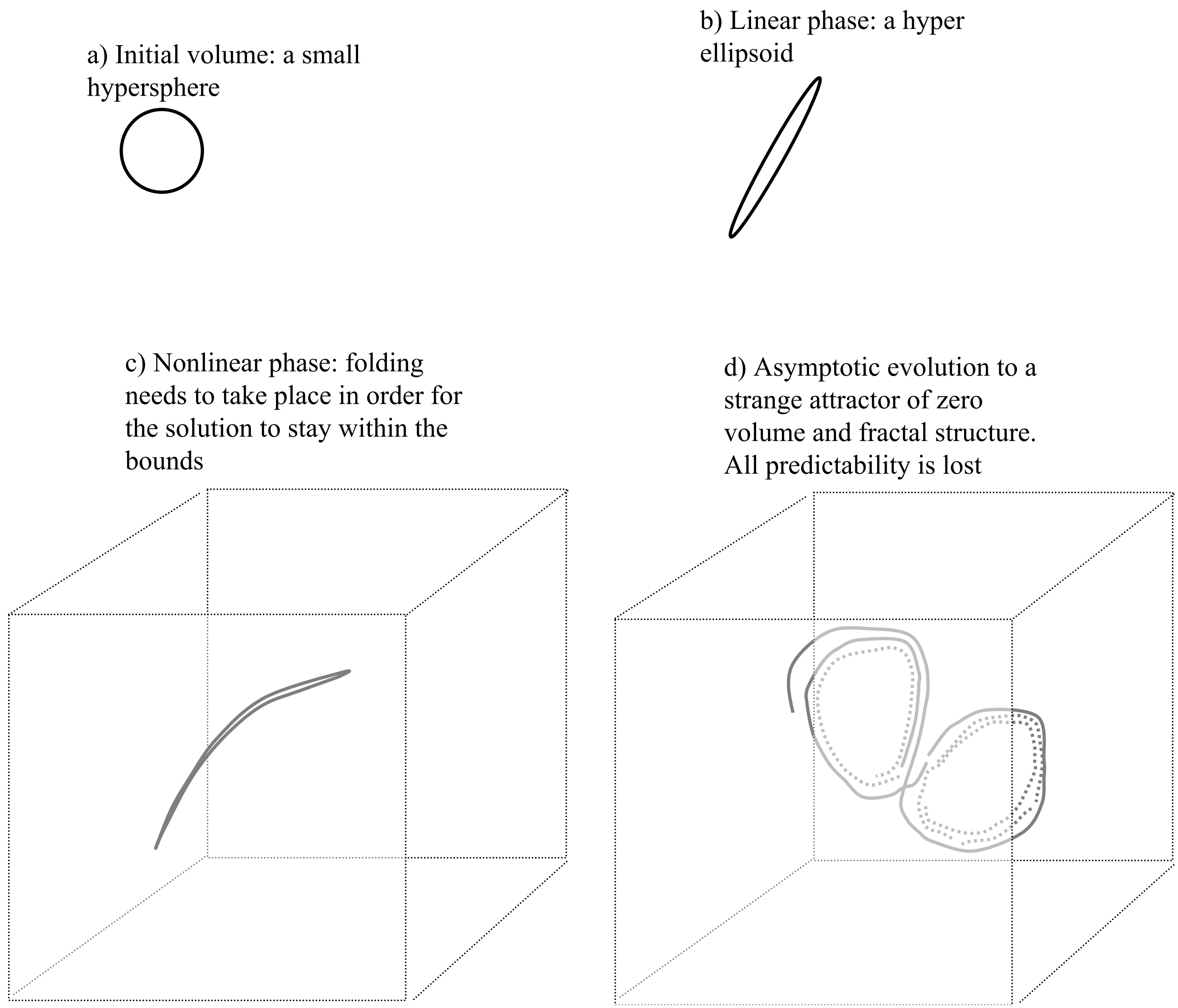


Predictability depends on the initial conditions (Palmer, 2002):



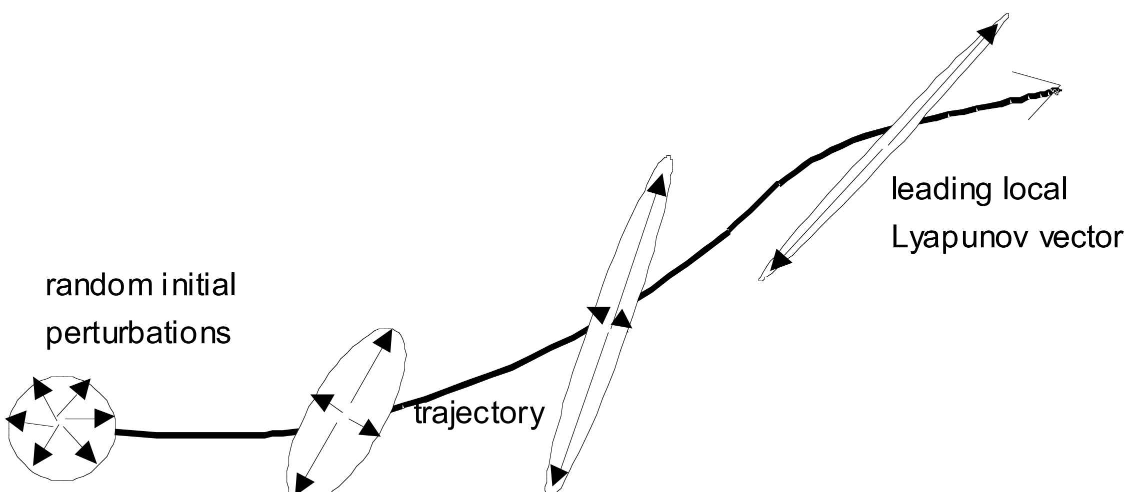
A "ball" of perturbed initial conditions is followed with time. Errors in the initial conditions that are unstable (with "errors of the day") grow much faster than if they are stable

Fig. 6.2: Schematic of the evolution of a small spherical volume in phase space in a bounded dissipative system.



When there is an instability, all perturbations converge towards the fastest growing perturbation (leading Lyapunov Vector). The LLV is computed applying the linear tangent model on each perturbation of the nonlinear trajectory

Fig. 6.7: Schematic of how all perturbations will converge towards the leading Local Lyapunov Vector



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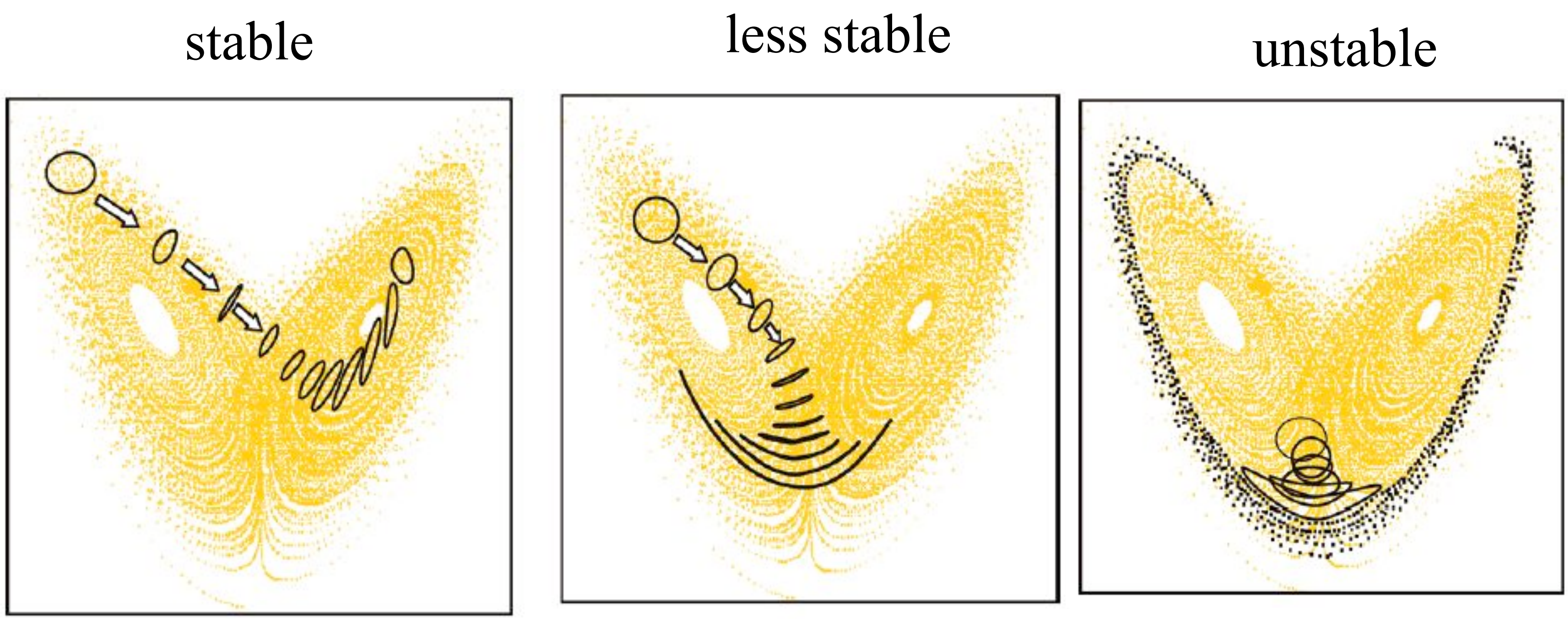
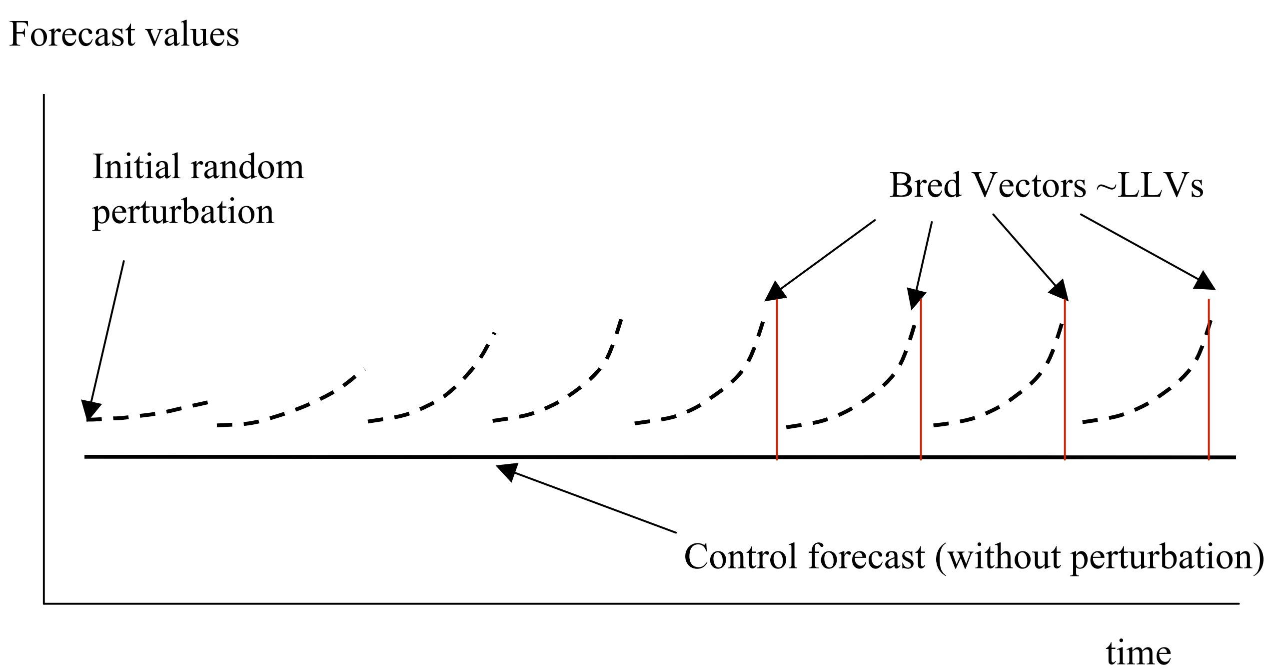


A “ball” of perturbed initial conditions is followed with time. Errors in the initial conditions that are unstable (with “errors of the day”) grow much faster than if they are stable

"Breeding": Grow naturally unstable perturbations, similar to Lyapunov vectors but using the nonlinear model twice

- Breeding is simply running the nonlinear model a second time, starting from perturbed initial conditions, rescaling the perturbation periodically

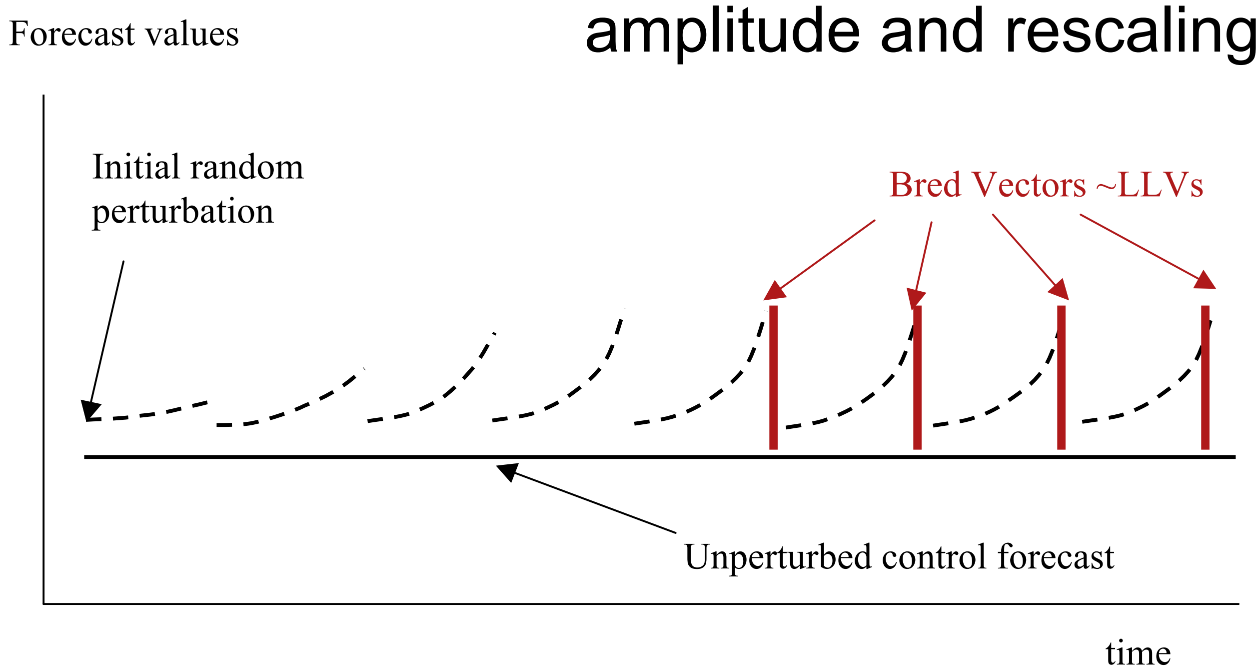


An 8 week RISE project for undergraduate women (2002)

- We gave a team of 4 RISE intern undergraduates a problem: Play with the famous Lorenz (1963) model, and explore its predictability using "breeding" (Toth and Kalnay 1993), a very simple method to study the growth of errors.
- We told them: "Imagine that you are forecasters that live in the Lorenz 'attractor'. Everybody living in the attractor knows that there are two weather regimes, the 'Warm' and 'Cold' regimes. But what the public needs to know is when will the change of regimes take place, and how long are they going to last!!".
- "Can you find a forecasting rule to alert the public that there is an imminent change of regime?"

Breeding: simply running the nonlinear model a second time, from perturbed initial conditions.

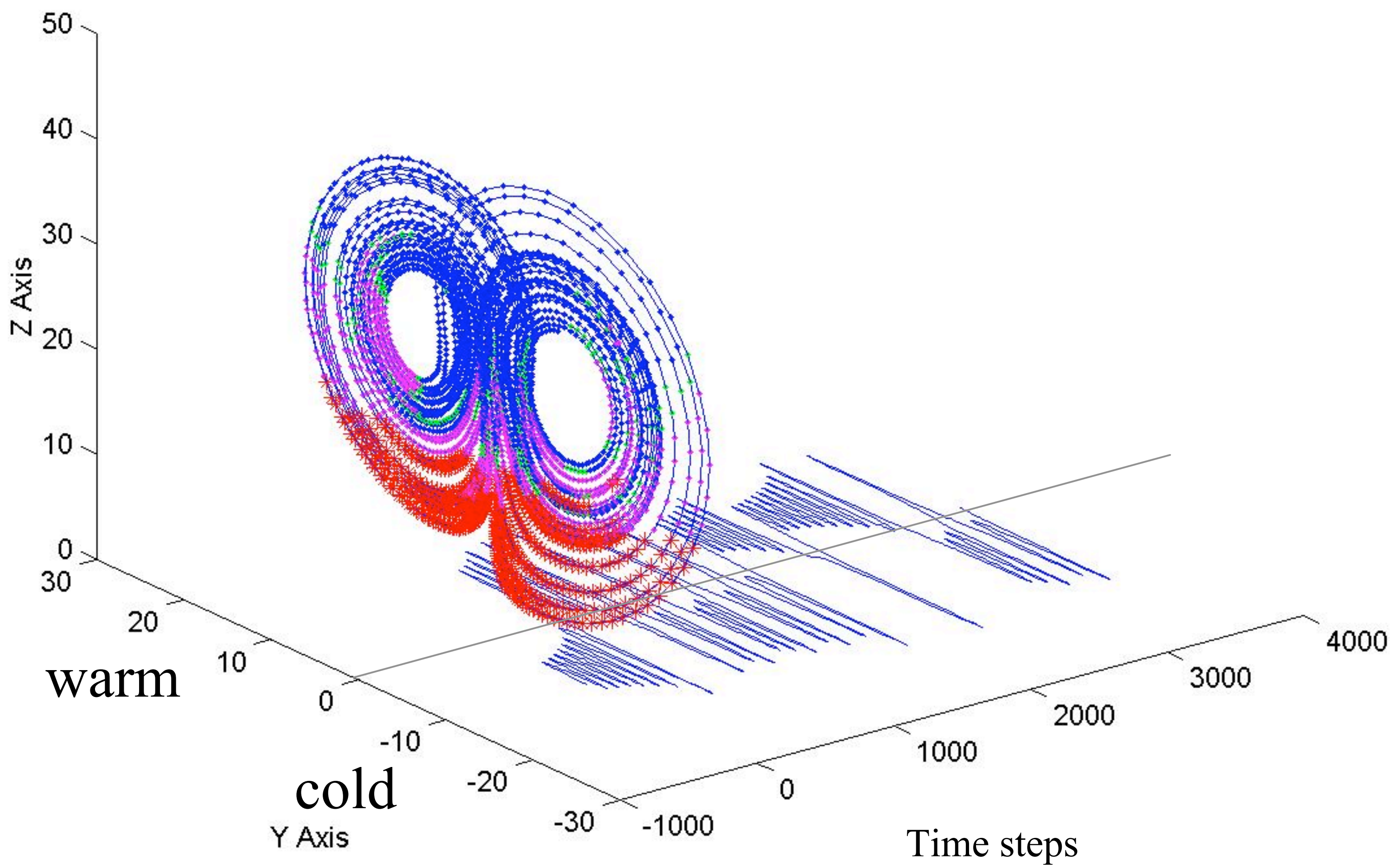
Only two tuning parameters: rescaling amplitude and rescaling interval



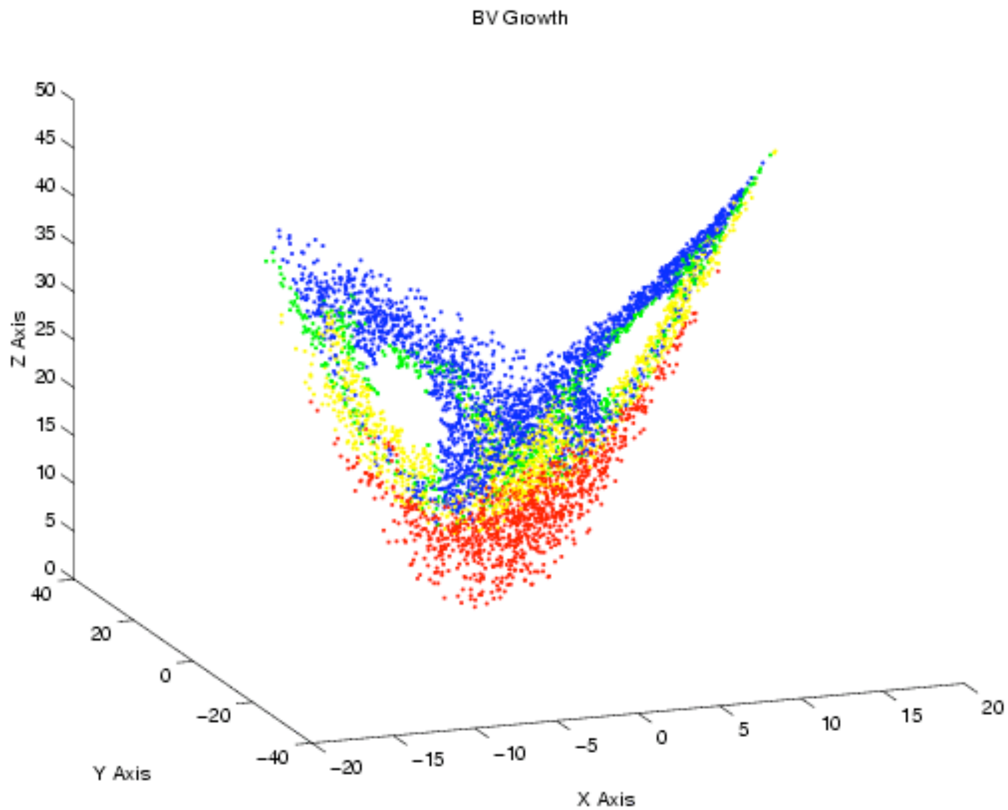
Local breeding growth rate:

$$g(t) = \frac{1}{n\Delta t} \ln\left(\left| \delta \mathbf{x} \right| / \left| \delta \mathbf{x}_0 \right| \right)$$

4 summer interns computed the Lorenz Bred Vector growth rate: red means large BV growth, blue means perturbations decay



In the 3-variable Lorenz (1963) model we used breeding to estimate the local growth of perturbations:

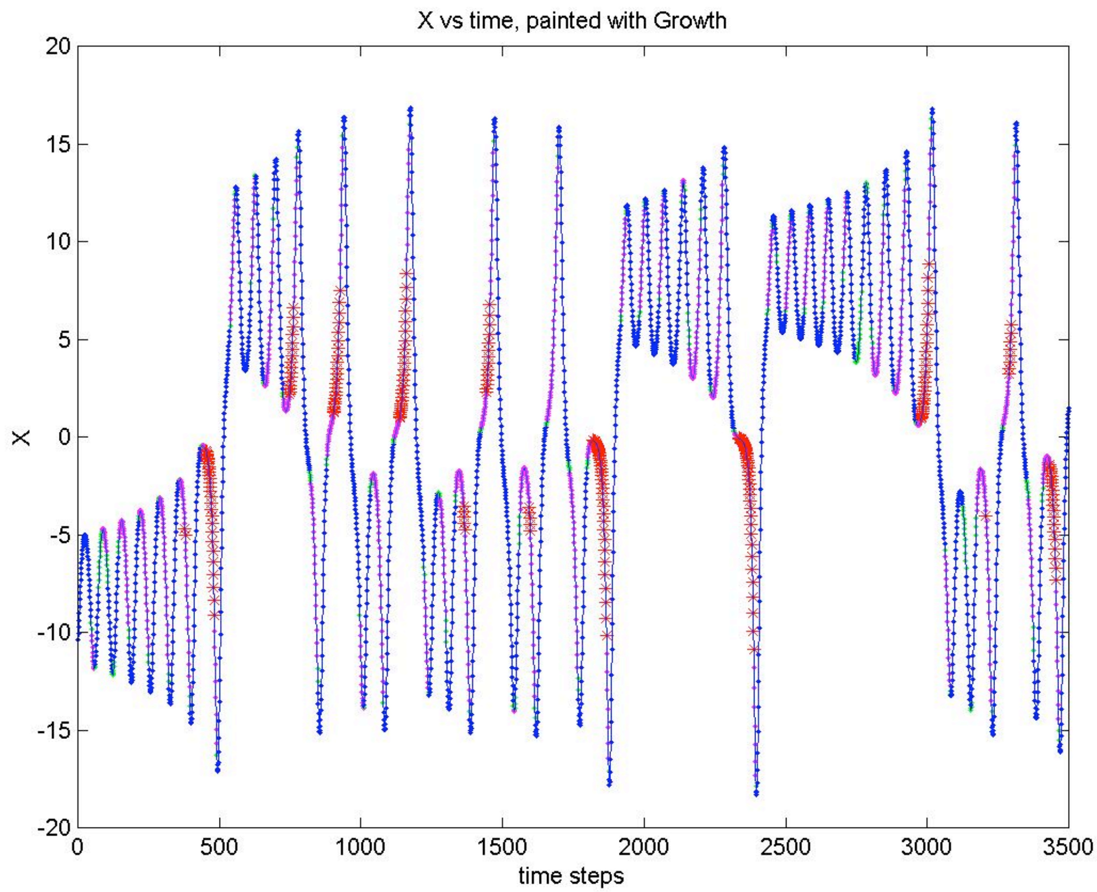


Bred Vector Growth:
red, high growth;
yellow, medium;
green, low growth;
blue, decay

With just a single breeding cycle, we can estimate the stability of the whole attractor (Evans et al, 2004)

This looked promising, so we asked the interns to "paint" x(t) with the bred vector growth, and the result almost made me faint:

This looked promising, so we asked the interns to "paint" x(t) with the bred vector growth, and the result almost made me faint:



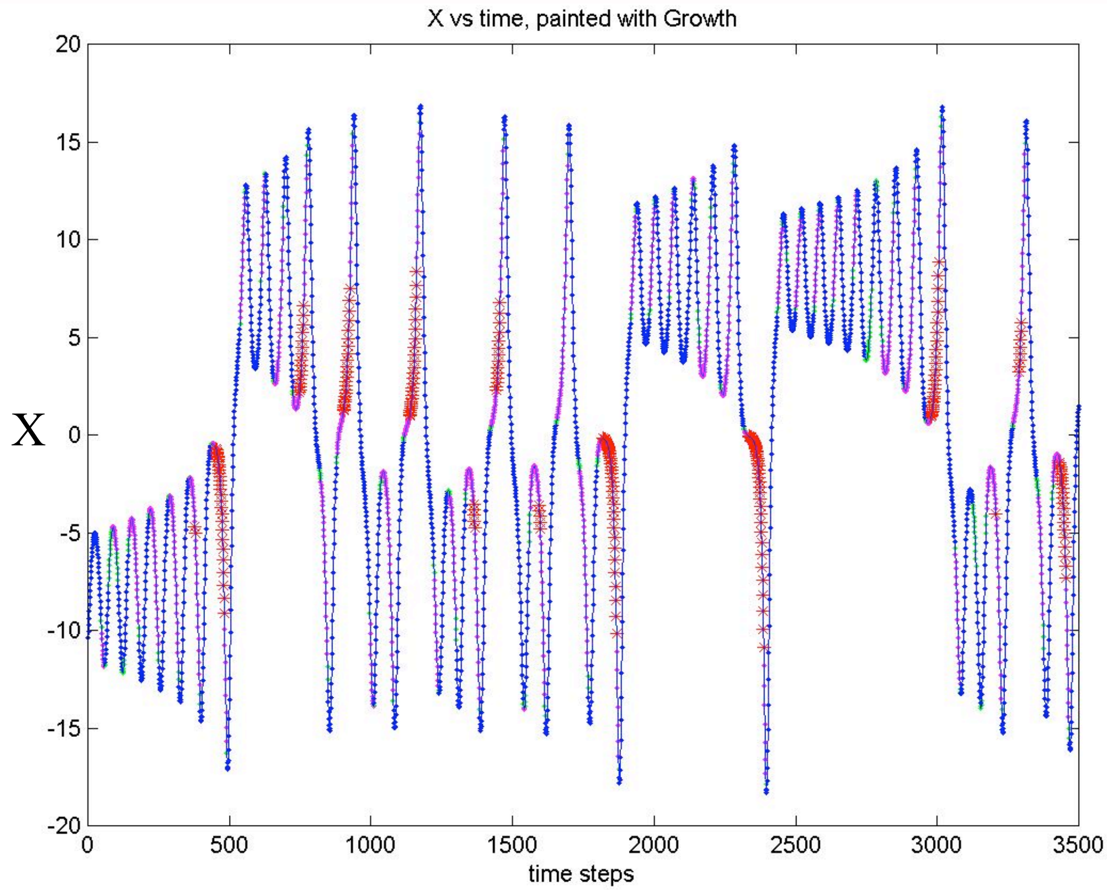
Growth rate of bred vectors:

A * indicates fast growth (>1.8 in 8 steps)

Forecasting rules for the Lorenz model:

“warm”

“cold”



Growth rate of bred vectors:

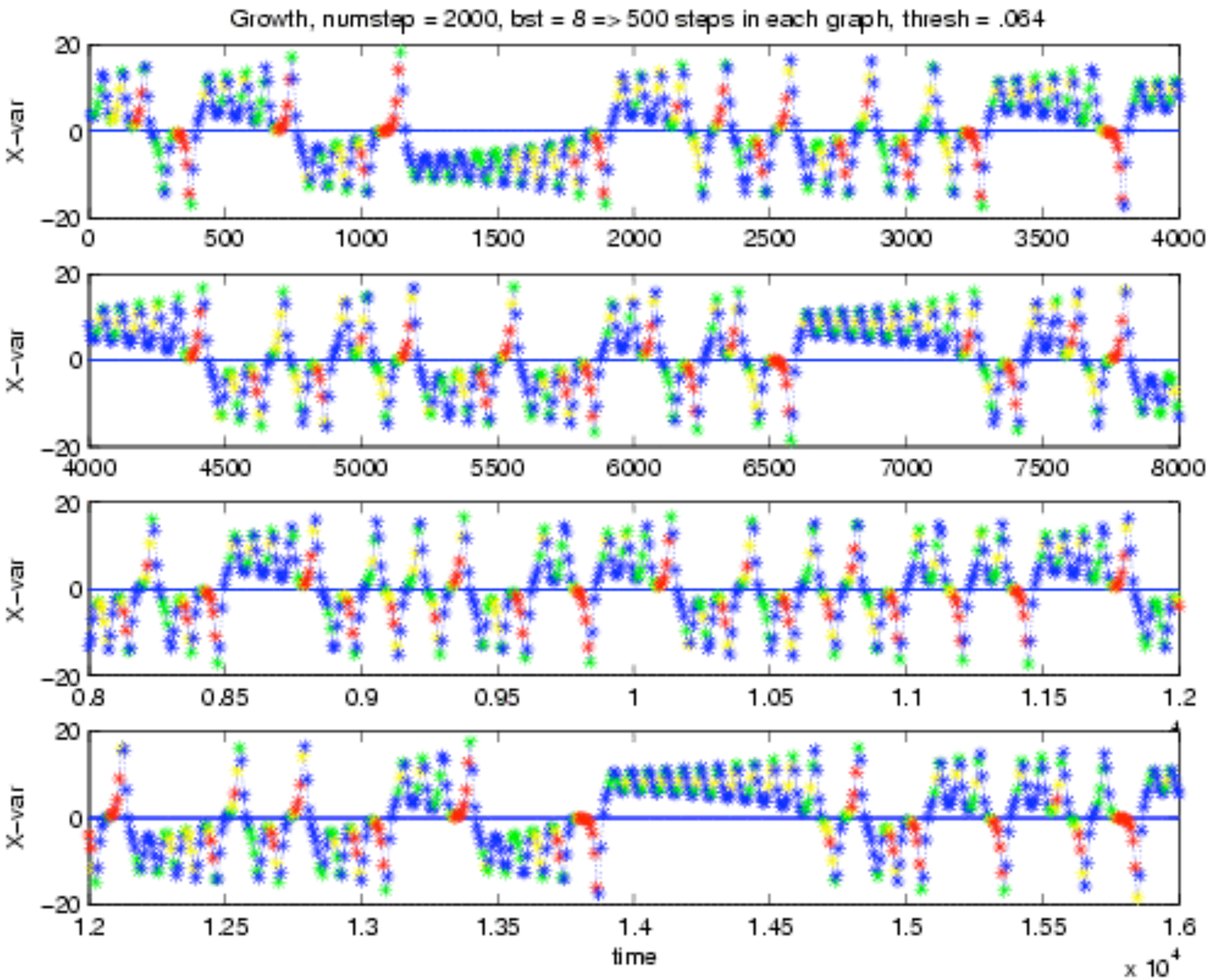
A * indicates fast growth (>1.8 in 8 steps)

Regime change:The presence of red stars (fast BV growth) indicates that the next orbit will be the last one in the present regime.

Regime duration: One or two red stars, next regime will be short. Several red stars: the next regime will be long lasting.

These rules surprised Lorenz himself!

These are very robust rules, with skill scores > 95%



Summary for this part and rest of the talk

- Breeding is a simple generalization of Lyapunov vectors, for finite time, finite amplitude: simply run the model twice, take the difference and rescale…
- Breeding in the Lorenz (1963) model gives accurate forecasting rules for the “chaotic” change of regime and duration of the next regime that surprised Lorenz!

Rest of the talks 1 and 3:

- The same ideas can be applied to fight chaos in the full forecast models that have dimension 10-100 million rather than just 3!
- In the atmosphere, in the ocean, and in coupled systems
- We can also use breeding to understand the **physical mechanisms** of the instabilities that create chaos

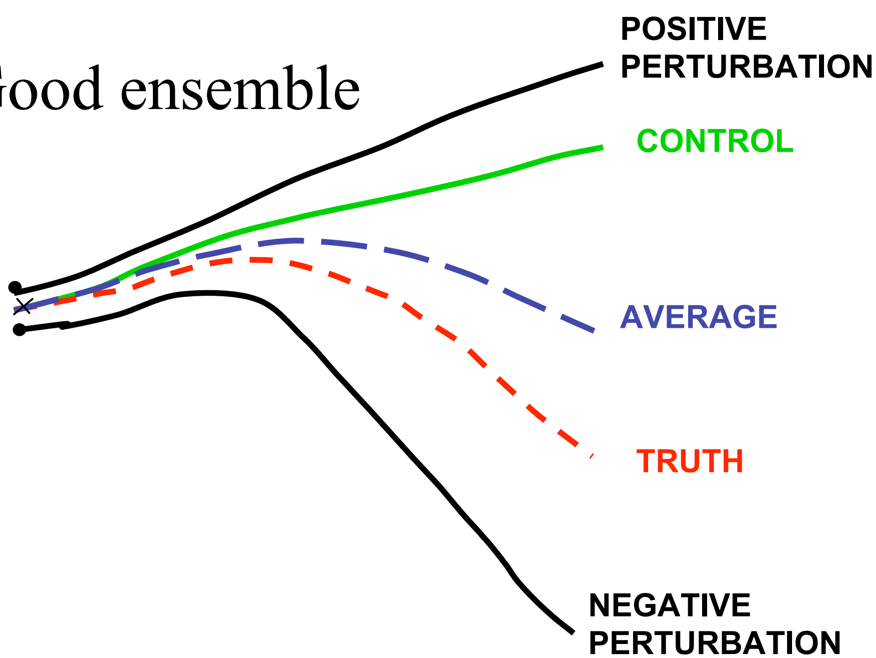
A major tool to "fight chaos" is ensemble forecasting

An ensemble forecast starts from initial perturbations to the analysis…

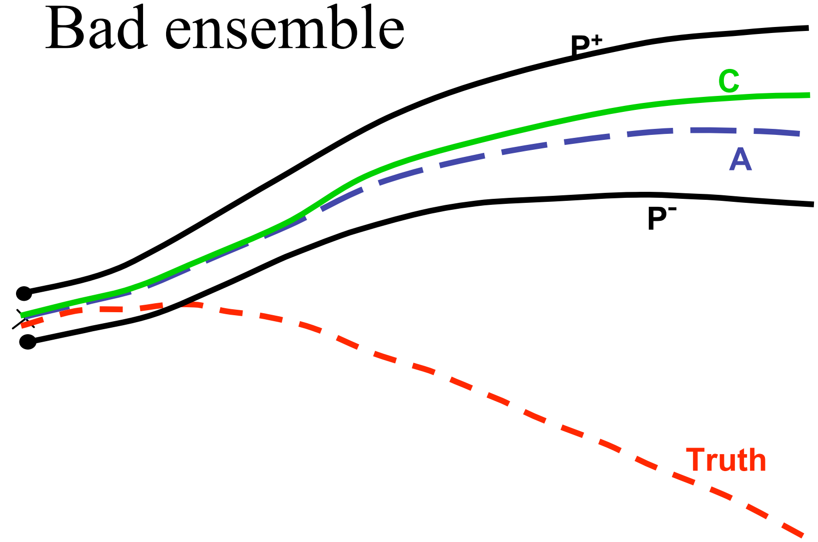
In a good ensemble "truth" looks like a member of the ensemble

The initial perturbations should reflect the analysis "errors of the day"

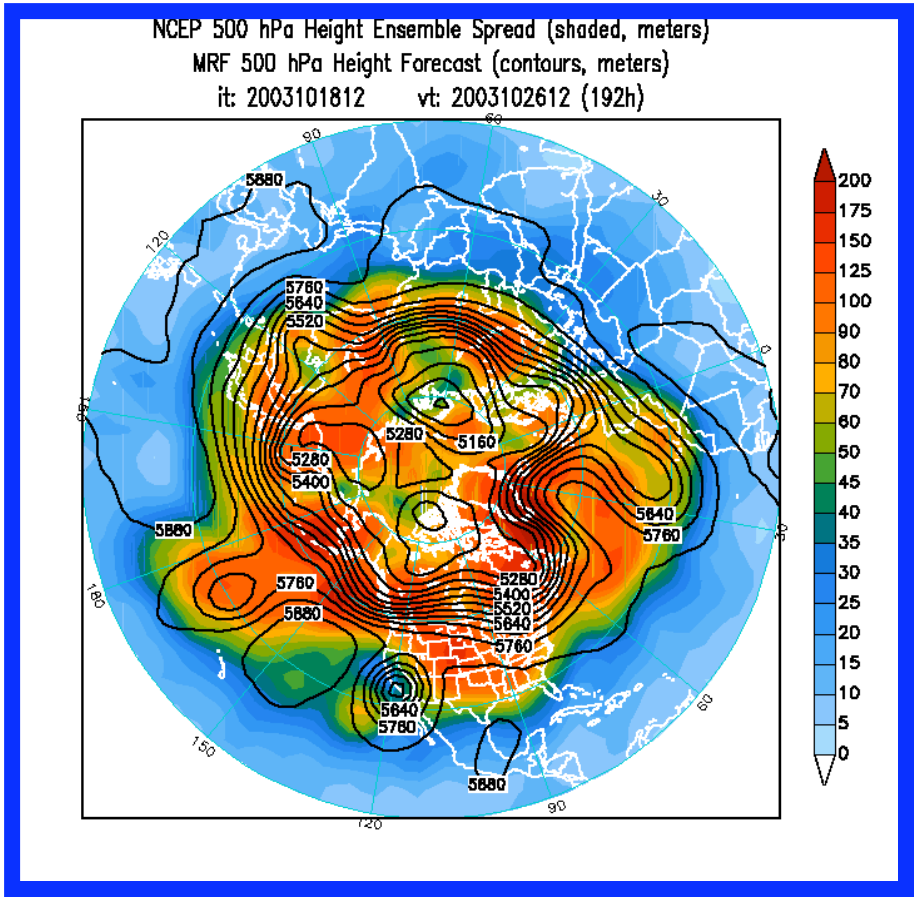
Good ensemble

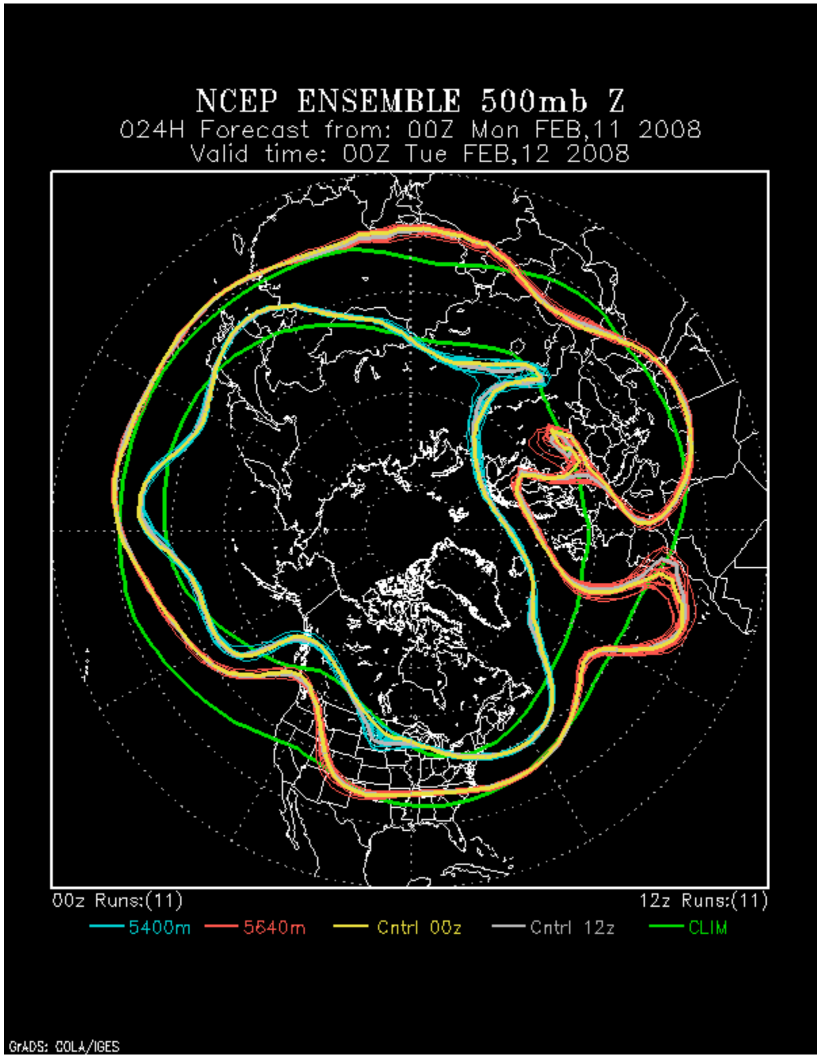


Bad ensemble



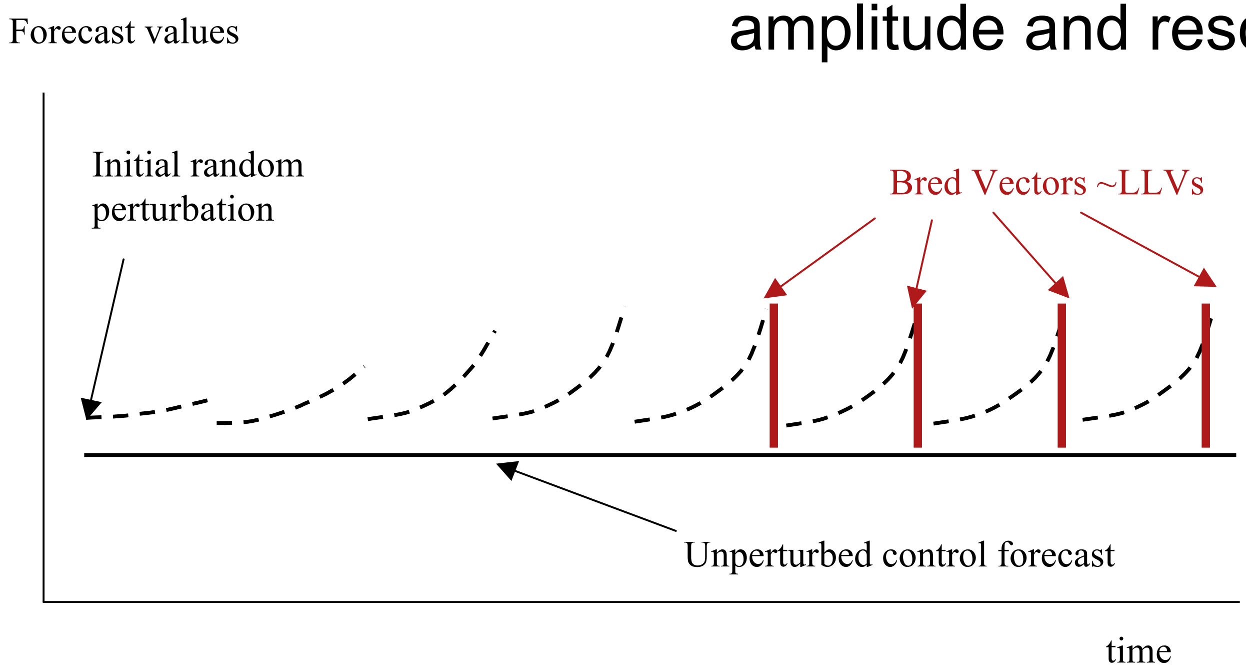
In ensemble forecasting we need to represent the uncertainty: spread or "spaghetti plots"





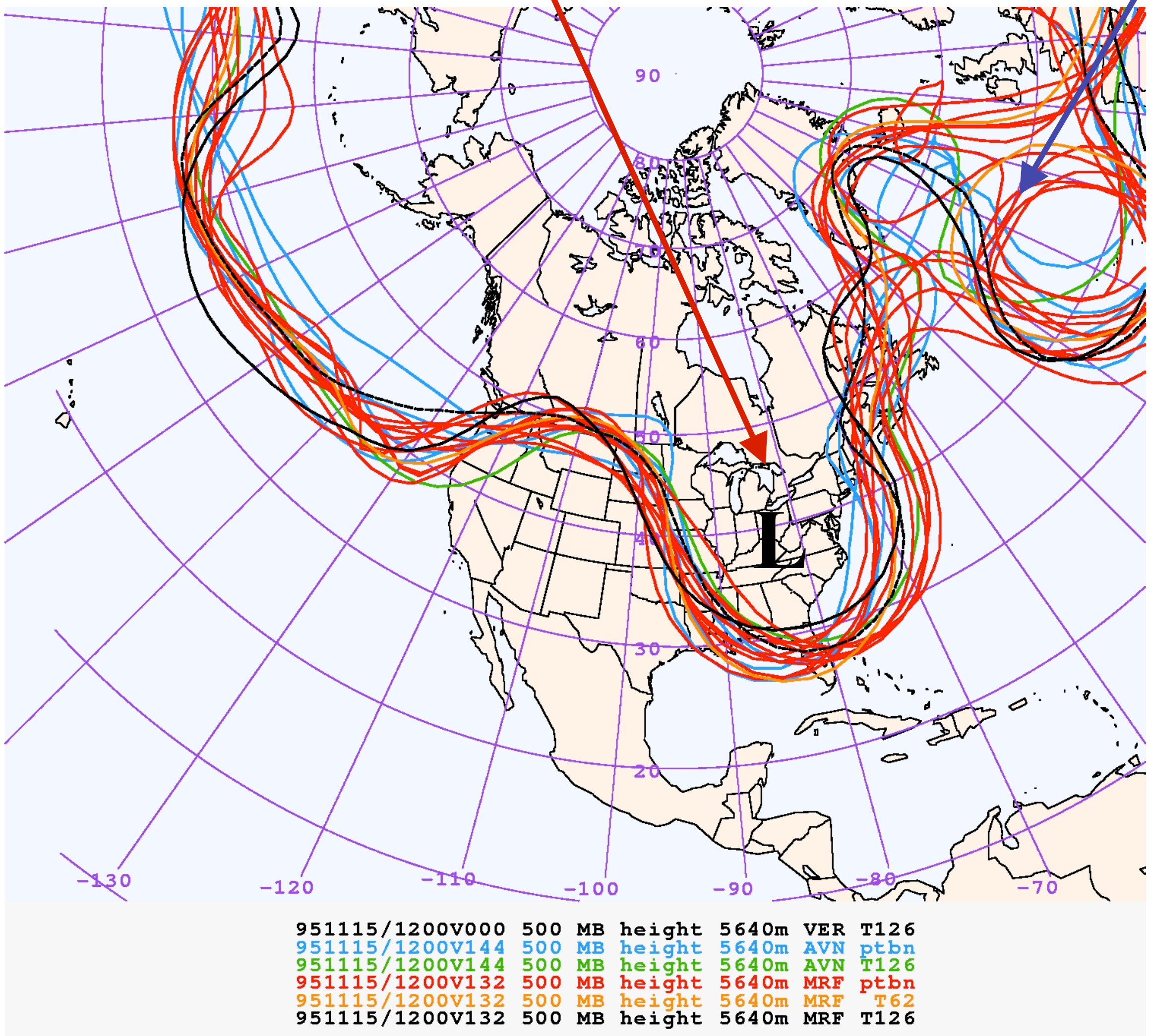
Breeding: running the nonlinear model again from perturbed initial conditions: introduced by Toth and Kalnay (1993) to create initial ensemble perturbations

Only two tuning parameters: rescaling amplitude and rescaling interval



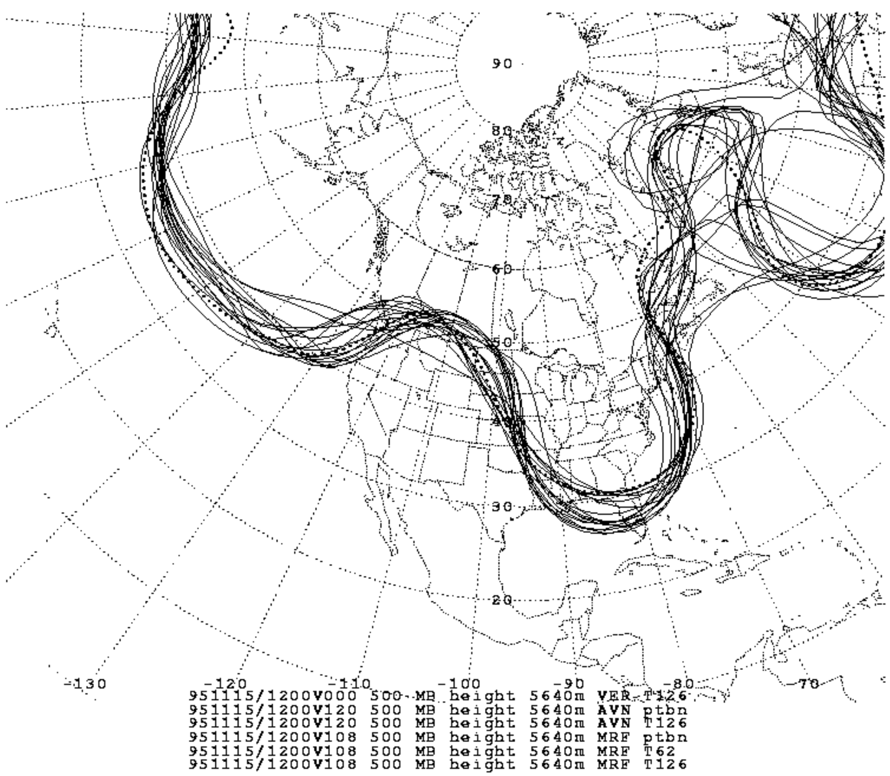
Local breeding growth rate: $$g(t) = \frac{1}{n\Delta t} \ln\left(|\delta \mathbf{x}| / |\delta \mathbf{x}_0| \right)$$

Example of a very predictable 6-day forecast, with "errors of the day"



The bred vectors are the growing atmospheric perturbations: "errors of the day"

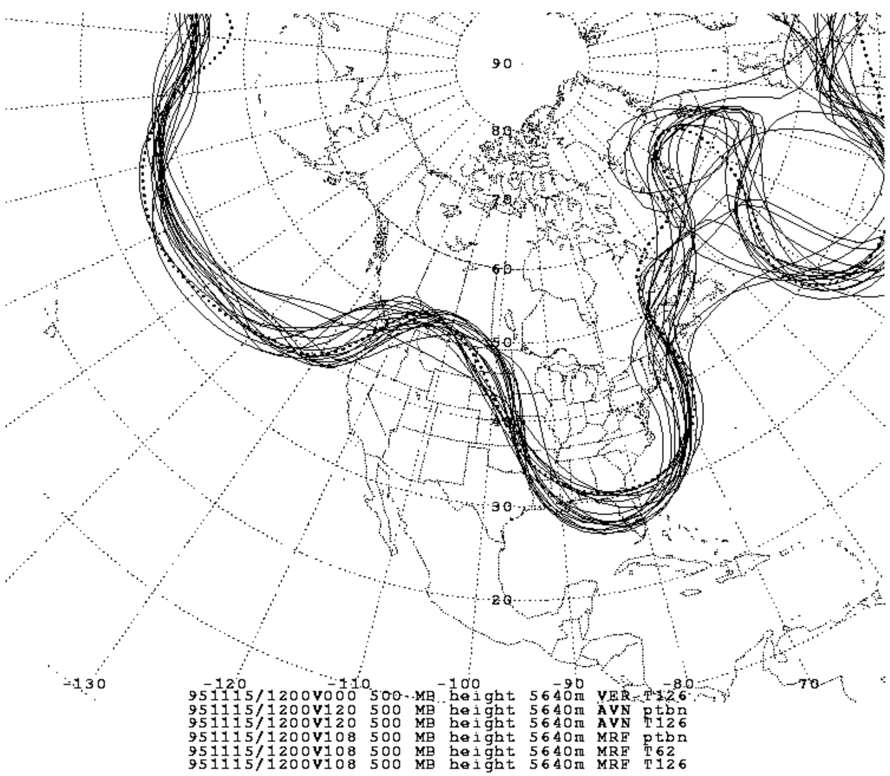
The errors of the day are instabilities of the background flow. At the same verification time, the forecast uncertainties have *the same shape*

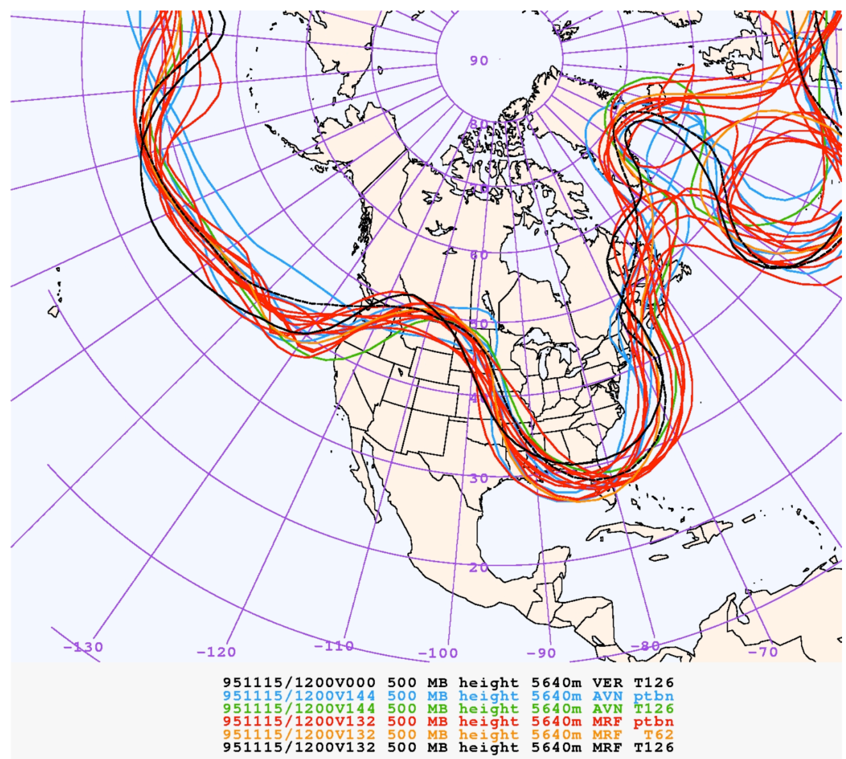


4-day forecast verifying on the same day

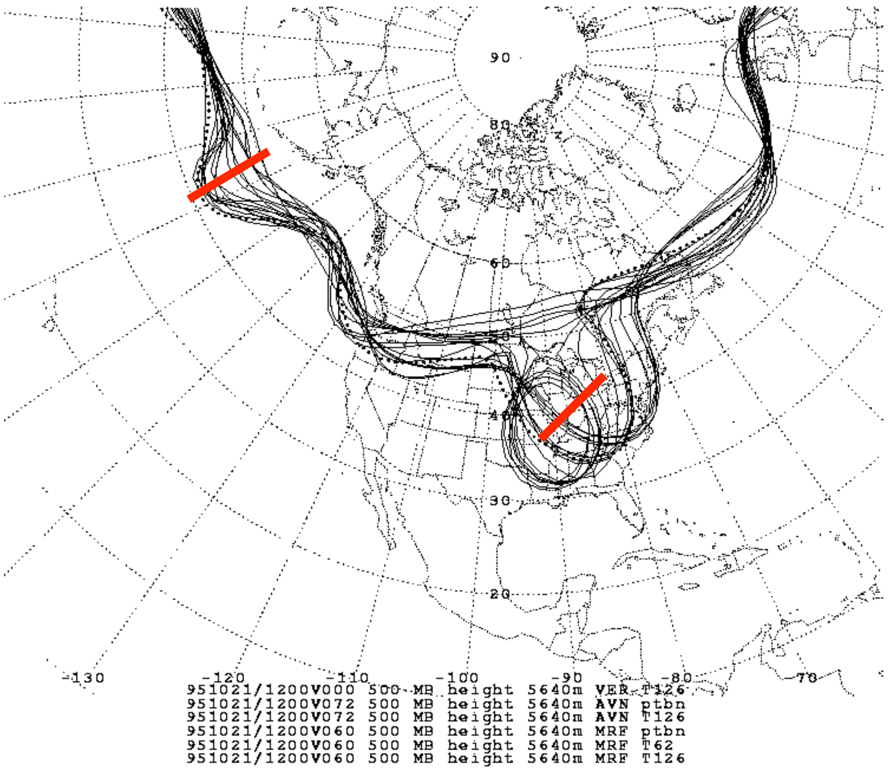
The errors of the day are instabilities of the background flow. At the same verification time, the forecast uncertainties have *the same shape*

4 days and 6 days ensemble forecasts verifying on 15 Nov 1995





Strong instabilities of the background tend to have simple shapes (perturbations lie in a low-dimensional subspace of bred vectors)



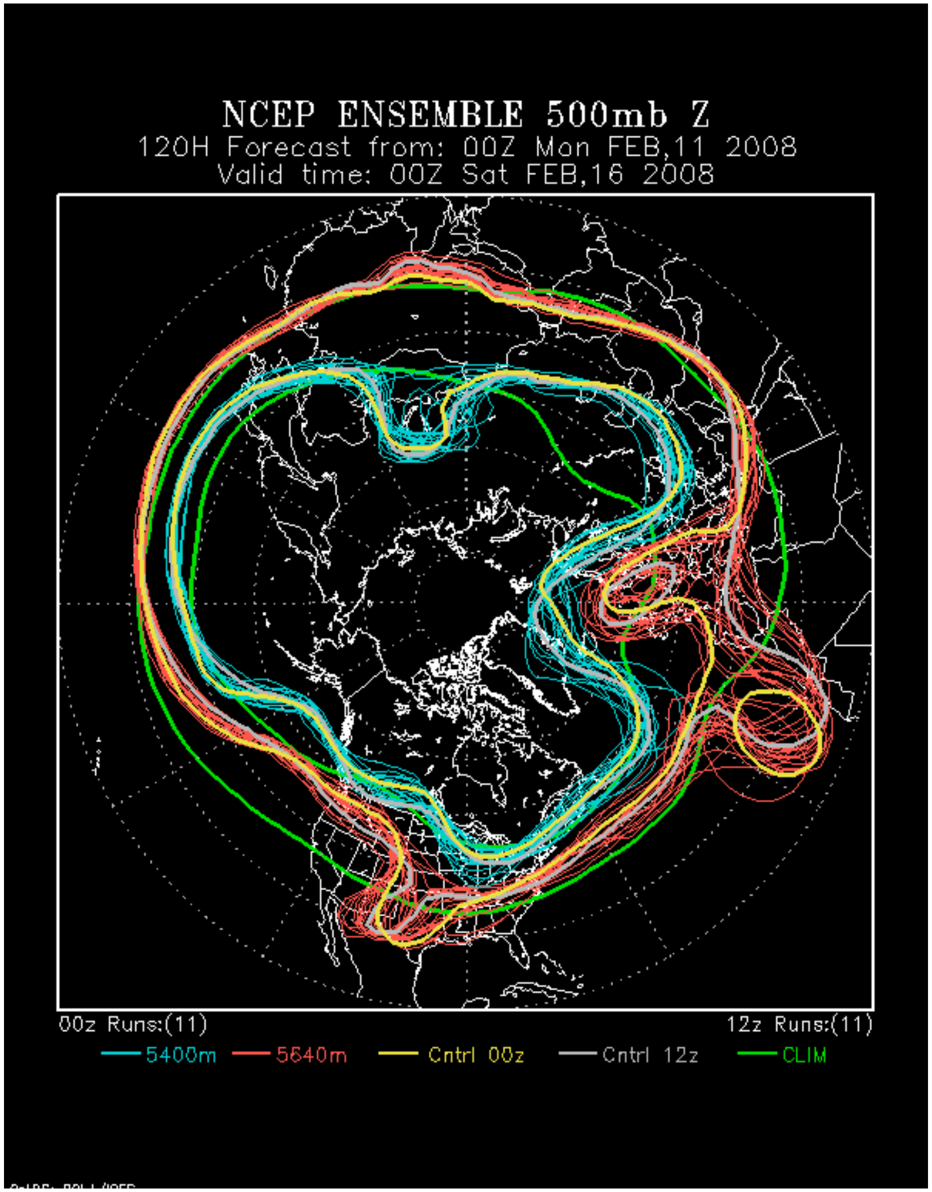
2.5 day forecast verifying on 95/10/21.

Note that the bred vectors (difference between the forecasts) lie on a 1-D space

This simplicity (**local low-dimensionality**, Patil et al. 2000) inspired the Local Ensemble Transform Kalman Filter (Ott et al. 2004, Hunt et al., 2007)

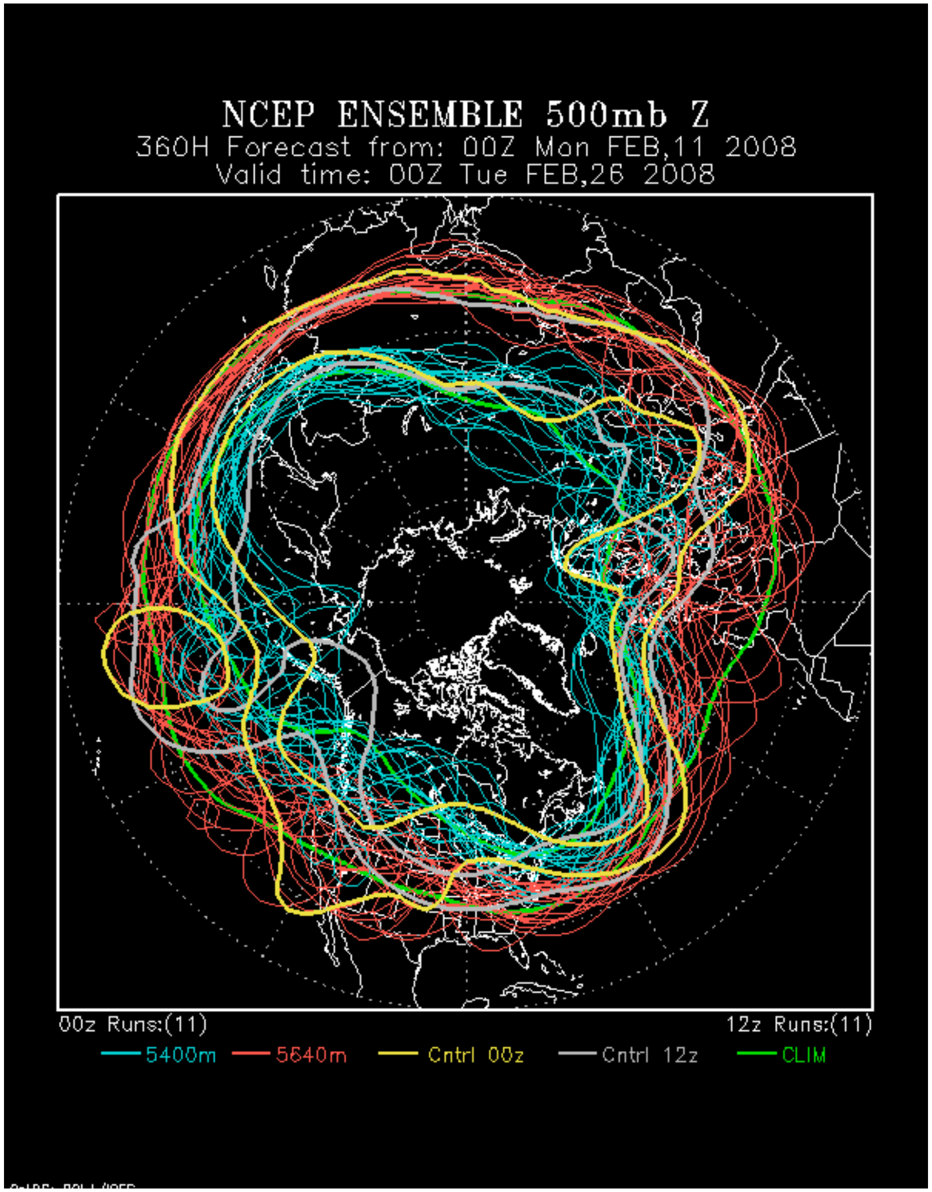
5-day forecast "spaghetti" plot

- The ensemble is able to separate the areas that are predictable from the ones that are chaotic.
- Even the chaotic ones have local low-dimensionality
- This is what makes possible to do Ensemble Kalman Filter with 50 ensemble members (not a million!) with good results



15-day forecast “spaghetti” plot: Chaos!

After 15 days, Lorenz’ chaos has won!
No predictability left in the 15-day forecast (except in East Asia)



Summary for this part

- Lorenz discovered the finite limit of predictability
- Predictability depends on the stability of the atmosphere: the errors of the day (or local Lyapunov vectors) that make model errors grow
- Ensemble forecasts allow us to estimate the predictability in space and in time

Elements of Ensemble Forecasting

- It used to be that a single control forecast was integrated from the analysis (initial conditions)
- In ensemble forecasting several forecasts are run from slightly perturbed initial conditions (or with different models)
- The spread among ensemble members gives information about the forecast errors
- How to create slightly perturbed initial conditions?
- Basically
  - Singular Vectors
  - Bred Vectors
  - Ensembles of data assimilation (perturbed obs. EnKF)

Summary for Lecture 1

- Lorenz discovered deterministic chaos
- Instabilities (“errors of the day”) make the atmosphere unpredictable beyond 2 weeks
- All perturbations evolve towards the most unstable (local Lyapunov vectors)
- Bred vectors are a finite time, nonlinear extension of LVs
- With ensemble forecasting, we fight chaos by estimating the local predictability in space and in time

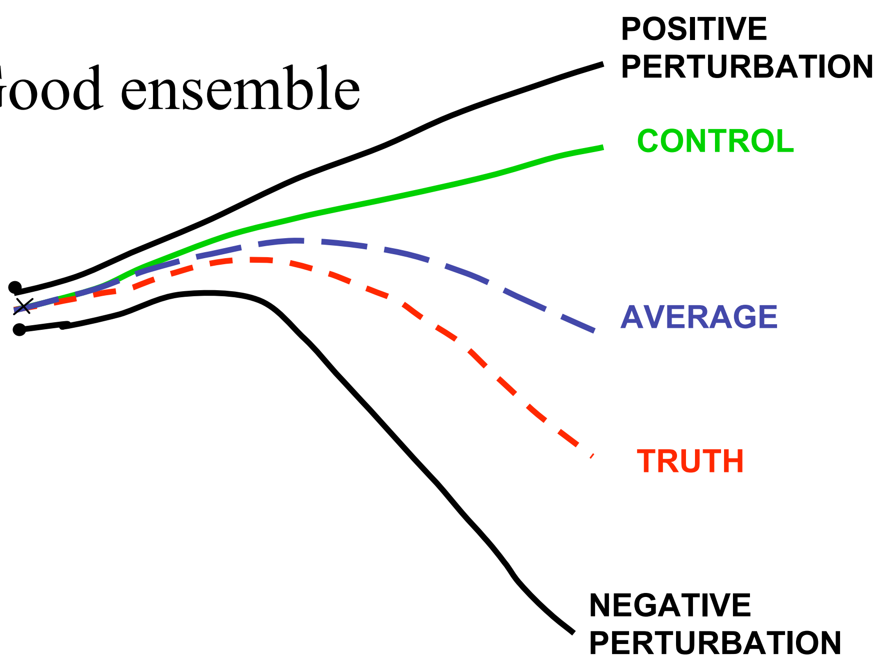
Components of ensemble forecasts

An ensemble forecast starts from initial perturbations to the analysis…

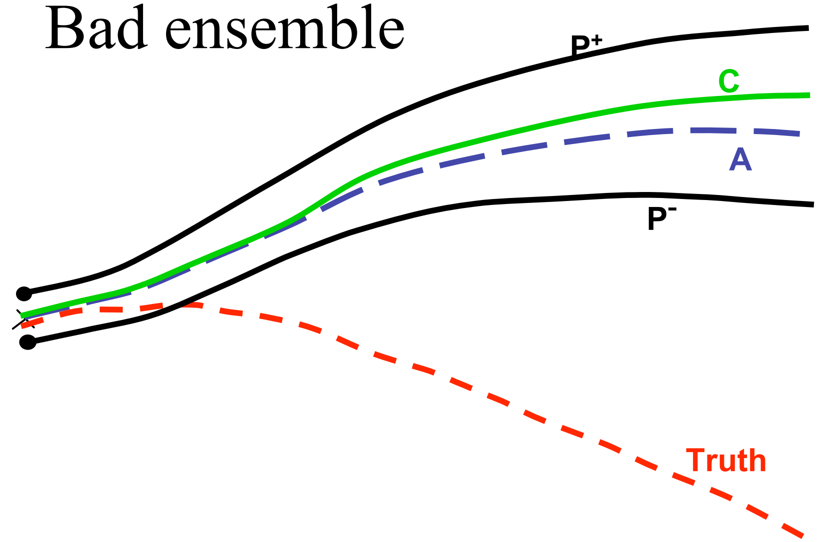
In a good ensemble "truth" looks like a member of the ensemble

The initial perturbations should reflect the analysis "errors of the day"

Good ensemble



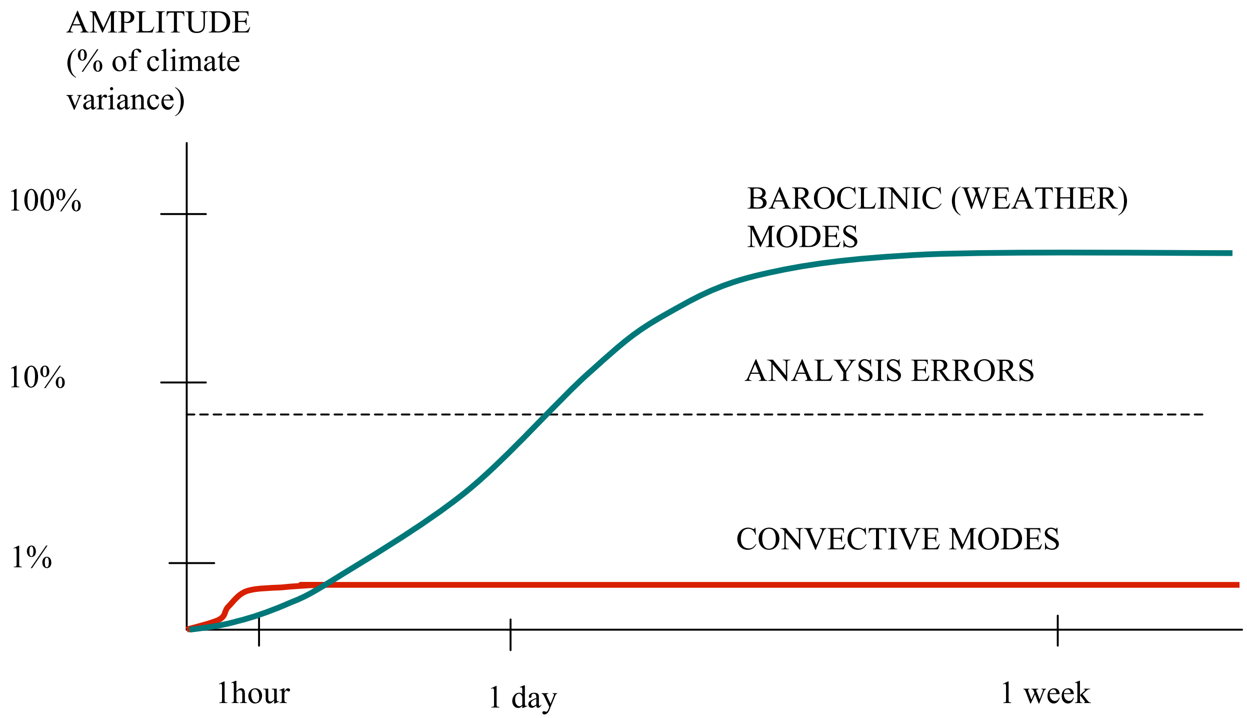
Bad ensemble



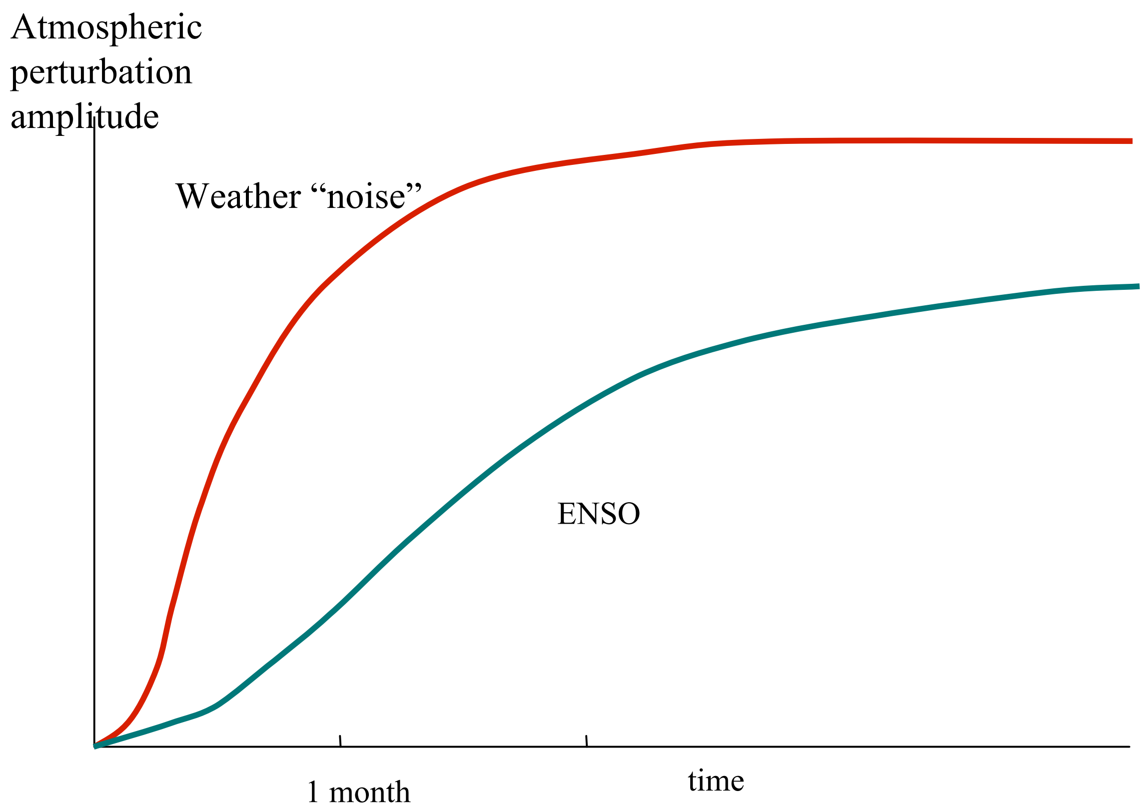
Data assimilation and ensemble forecasting in a coupled ocean-atmosphere system

- A coupled ocean-atmosphere system contains growing instabilities with many different time scales
  - The problem is to isolate the slow, coupled instability related to the ENSO variability.
- Results from breeding in the Zebiak and Cane model (Cai et al., 2002) demonstrated that
  - The dominant bred mode is the slow growing instability associated with ENSO
  - The breeding method has potential impact on ENSO forecast skill, including postponing the error growth in the “spring barrier”.
- Results from breeding in a coupled Lorenz model show that using amplitude and rescaling intervals chosen based on time scales, breeding can be used to separate slow and fast solutions in a coupled system.

Nonlinear saturation allows filtering unwanted fast, small amplitude, growing instabilities like convection (Toth and Kalnay, 1993)



In the case of coupled ocean-atmosphere modes, we cannot take advantage of the small amplitude of the "weather noise"! We can only use the fact that the coupled ocean modes are slower…



We coupled a slow and a fast Lorenz (1963) 3-variable model

Fast equations

$$\frac{dx_1}{dt} = \sigma(y_1 - x_1) - C_1(Sx_2 + O)$$

$$\frac{dy_1}{dt} = rx_1 - y_1 - x_1 z_1 + C_1(Sy_2 + O)$$

$$\frac{dz_1}{dt} = x_1 y_1 - bz_1 + C_1(Sz_2)$$

Slow equations

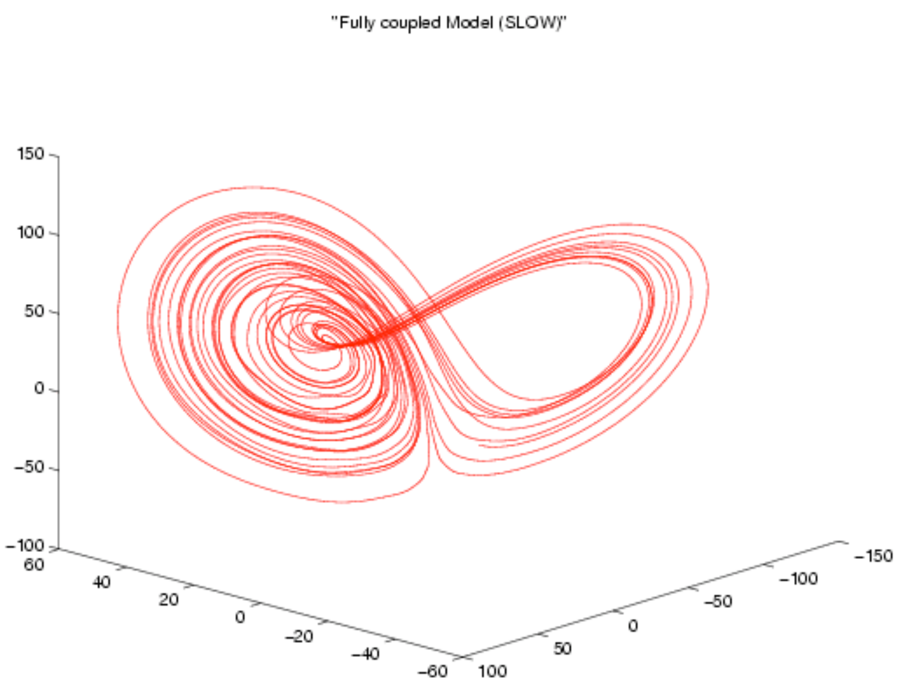
$$\frac{1}{\tau}\frac{dx_2}{dt} = \sigma(y_2 - x_2) - C_2(x_1 + O)$$

$$\frac{1}{\tau}\frac{dy_2}{dt} = rx_2 - y_2 - Sx_2 z_2 + C_2(y_1 + O)$$

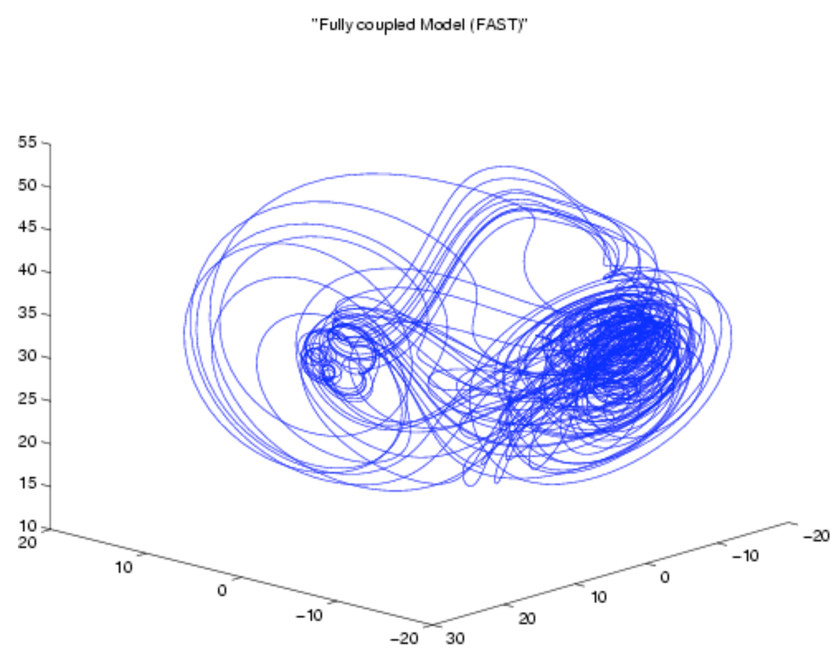
$$\frac{1}{\tau}\frac{dz_2}{dt} = Sx_2 y_2 - bz_2 + C_2(z_1)$$

Now we test the fully coupled “ENSO-like” system,
with similar amplitudes between “slow signal” and “fast noise”

“slow ocean”

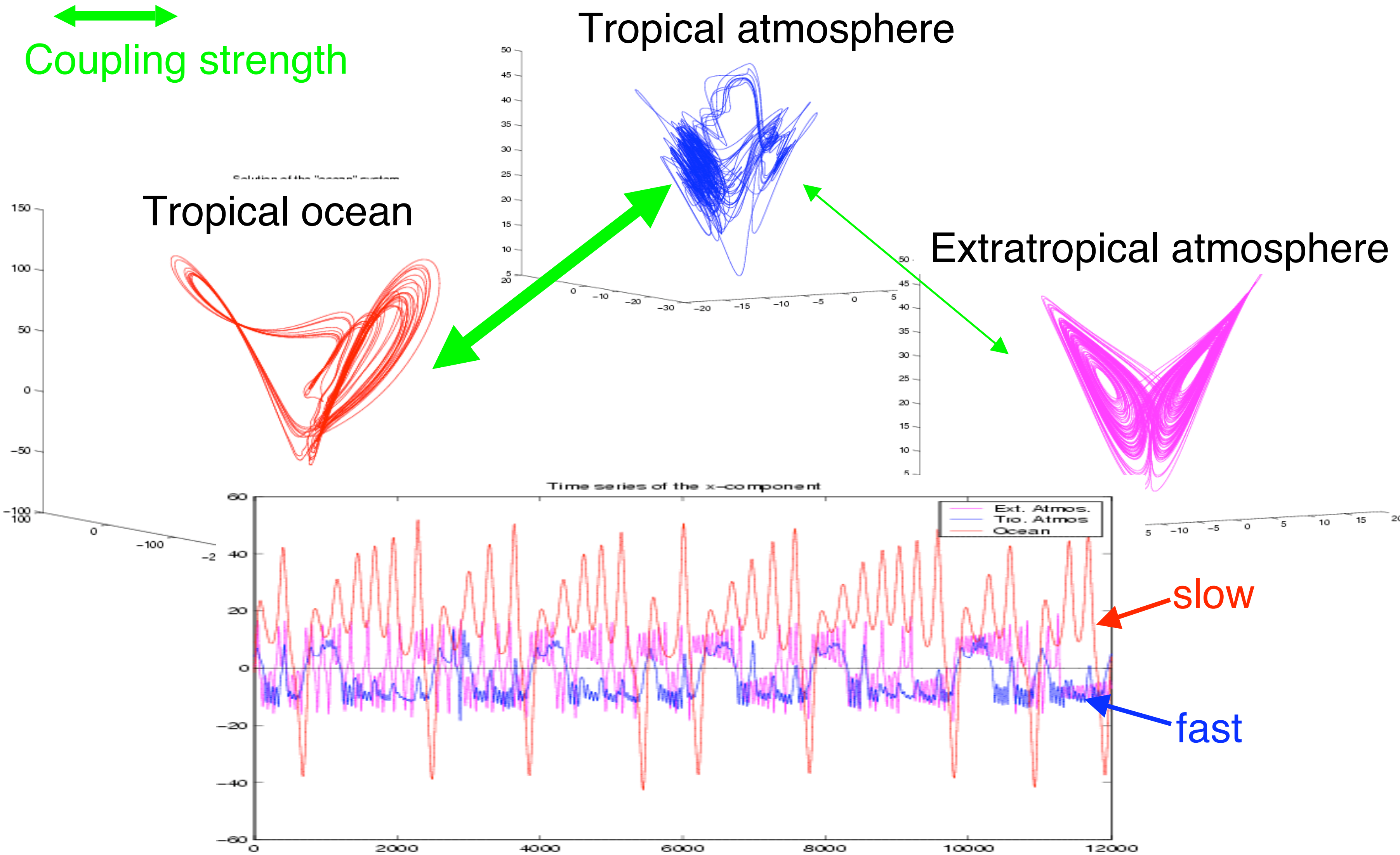


“tropical atmosphere”

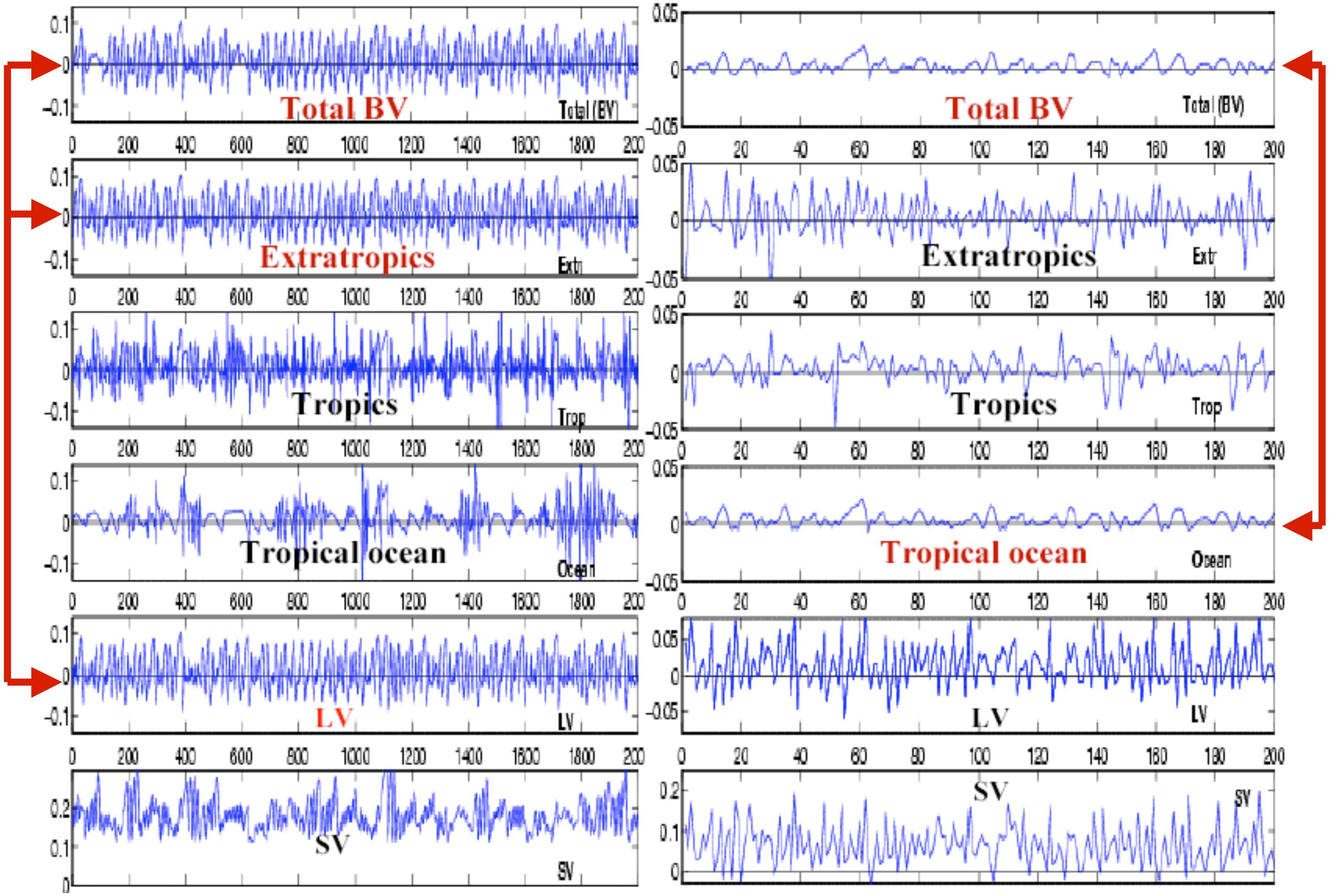


Then we added an extratropical atmosphere coupled with the tropics

Coupled fast and slow Lorenz 3-variable models (Peña and Kalnay, 2004)



Breeding in a coupled Lorenz model



Short rescaling interval (5 steps) and small amplitude: fast modes

Long rescaling interval (50 steps) and large amplitude: ENSO modes

The linear approaches (LV, SV) cannot capture the slow ENSO signal

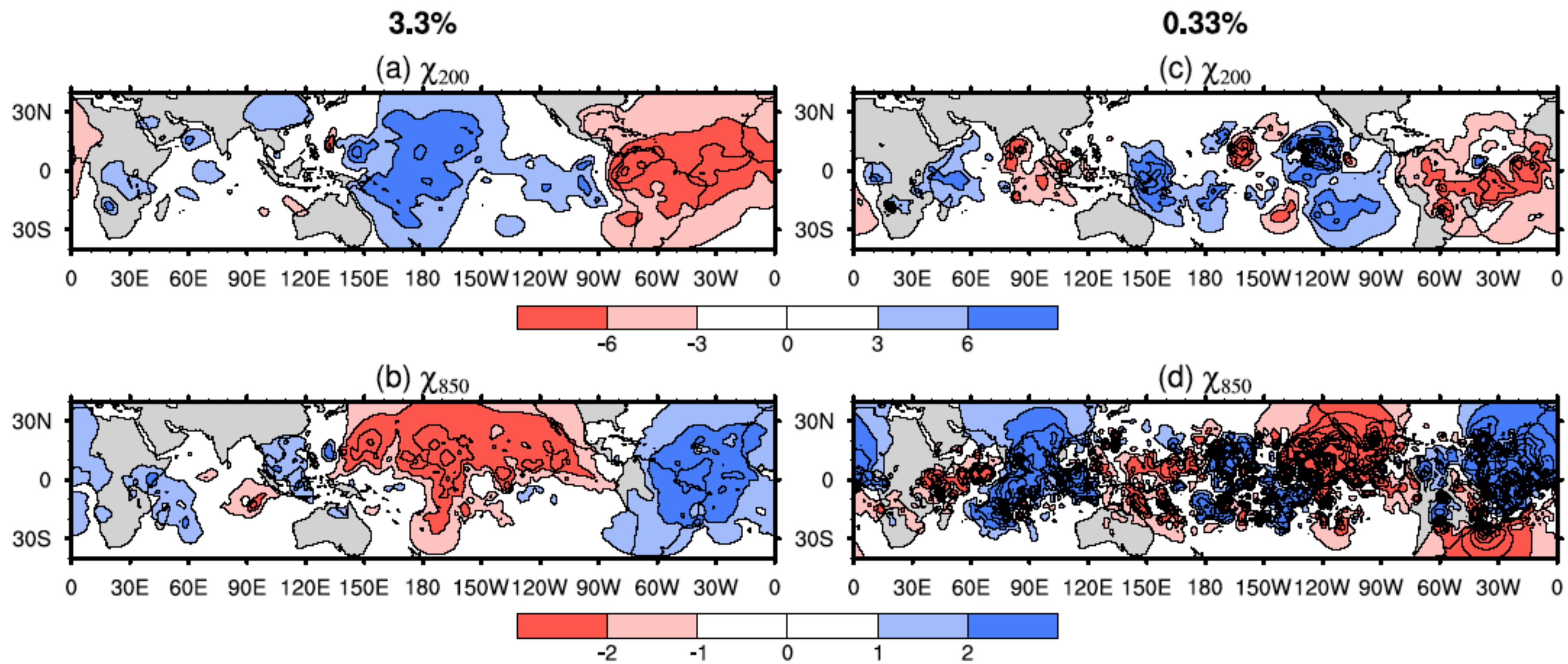
From Lorenz coupled models:

- In coupled fast/slow models, we can do breeding to isolate the slow modes
- We have to choose a slow variable and a long interval for the rescaling
- This is true for nonlinear approaches (e.g., EnKF) but not for linear approaches (e.g., SVs, LVs)
- This has been applied to ENSO coupled instabilities:
  - Cane-Zebiak model (Cai et al, 2003)
  - **NASA and NCEP fully coupled GCMs (Yang et al, 2006)**
  - **NASA operational system with real observations** (Yang et al. 2008)

Examples of breeding in a coupled ocean-atmosphere system with coupled instabilities

- In coupled fast/slow models, we can do breeding to isolate the slow modes
- We have to choose a slow variable and a long interval for the rescaling
- This identifies coupled instabilities.
- Examples
  - Madden-Julian Bred Vectors
  - NASA operational system with real observations (Yang et al 2007, MWR)
  - Ocean instabilities and their physical mechanisms (Hoffman et al, 2008, with thanks to Istvan Szunyogh)

Chikamoto et al (2007, GRL): They found the Madden-Julian instabilities BV by choosing an appropriate rescaling amplitude (only within the tropics)

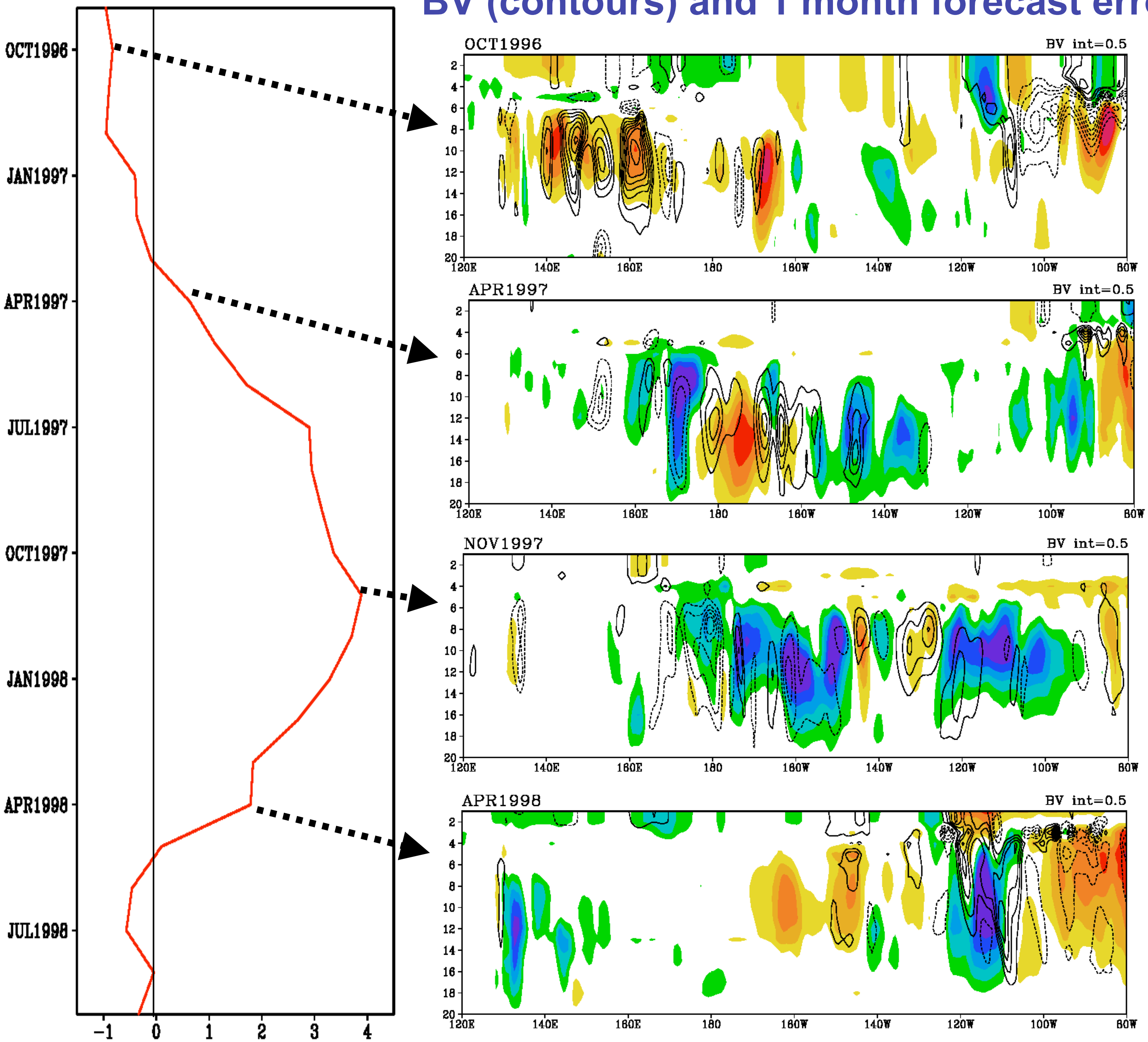


Finding the shape of the errors in El Niño forecasts to improve data assimilation

- **Bred vectors**:
  - Differences between the control forecast and perturbed runs:
  - **Should show the shape of growing errors**
- **Advantages**
  - Low computational cost (two runs)
  - Capture coupled instabilities
  - Improve data assimilation

Niño3 index

Yang (2005): Vertical cross-section at Equator for BV (contours) and 1 month forecast error (color)



Before 97’ El Niño, error is located in W. Pacific and near coast region

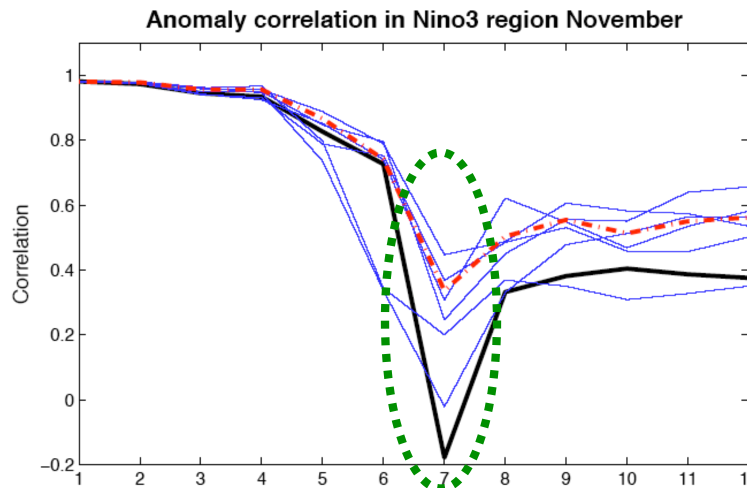
During development, error shifts to lower levels of C. Pacific.

At mature stage, error shifts further east and it is smallest near the coast.

After the event, error is located mostly in E. Pacific.

Yang: Impact of forecasts of El Niño with 3 pairs of BVs: November and May restarts (1993-2002)

Start from cold season

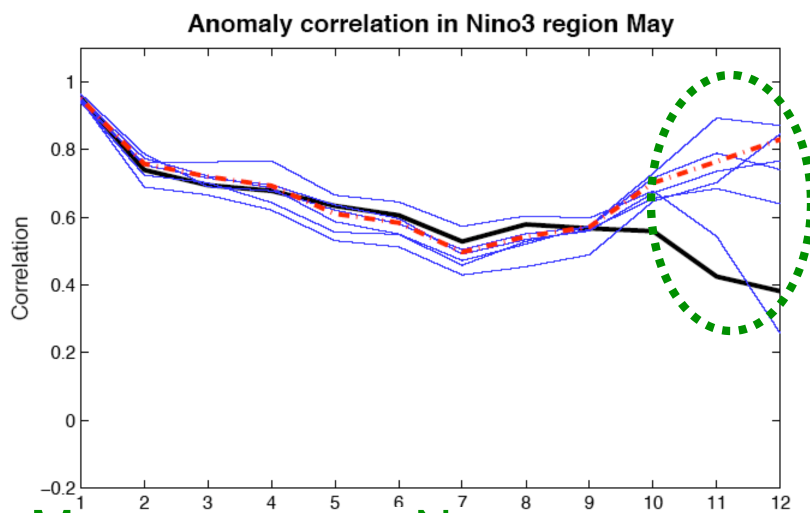


Nov May Forecast month

Control

BV ensemble mean

Start from warm season



May Nov Forecast month

BV ensemble improves upon the control "Spring barrier" loss of skill

Yang et al., 2006: Bred Vectors (contours) overlay Tropical Instability waves (SST): making them grow and break!