Bred vectors: theory and applications in operational forecasting.

Eugenia Kalnay

Lecture 3

Alghero, May 2008

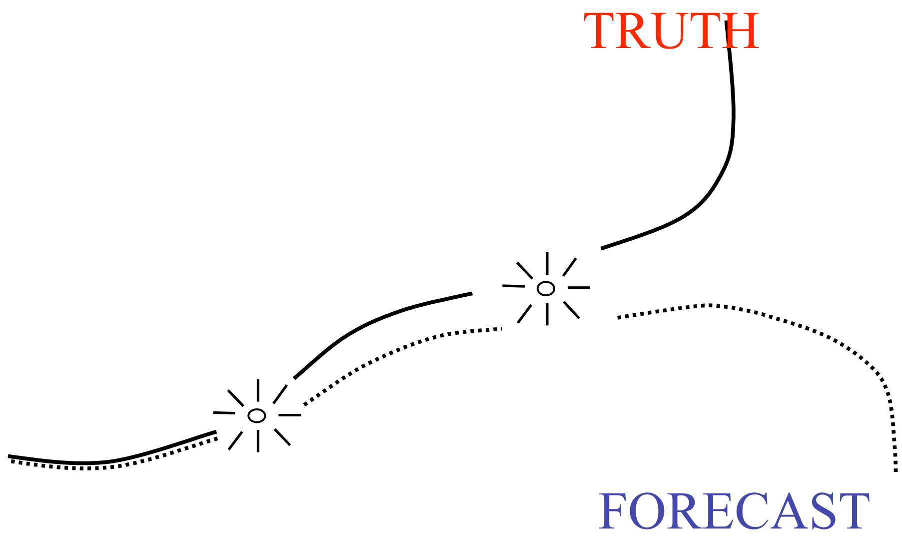

ca. 1974

Central theorem of chaos (Lorenz, 1960s):

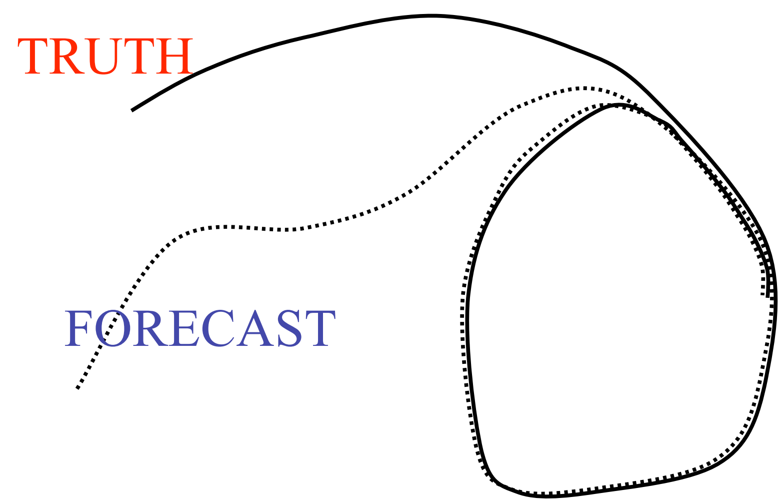
a) Unstable systems have finite predictability (chaos)

b) Stable systems are infinitely predictable

a) Unstable dynamical system



b) Stable dynamical system



Central theorem of chaos (Lorenz, 1960s):

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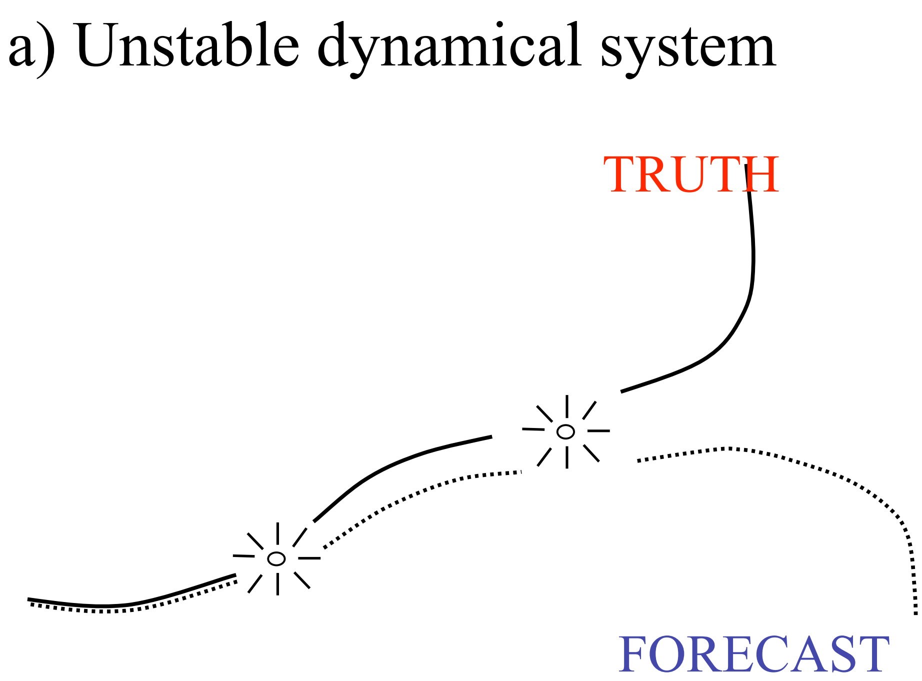
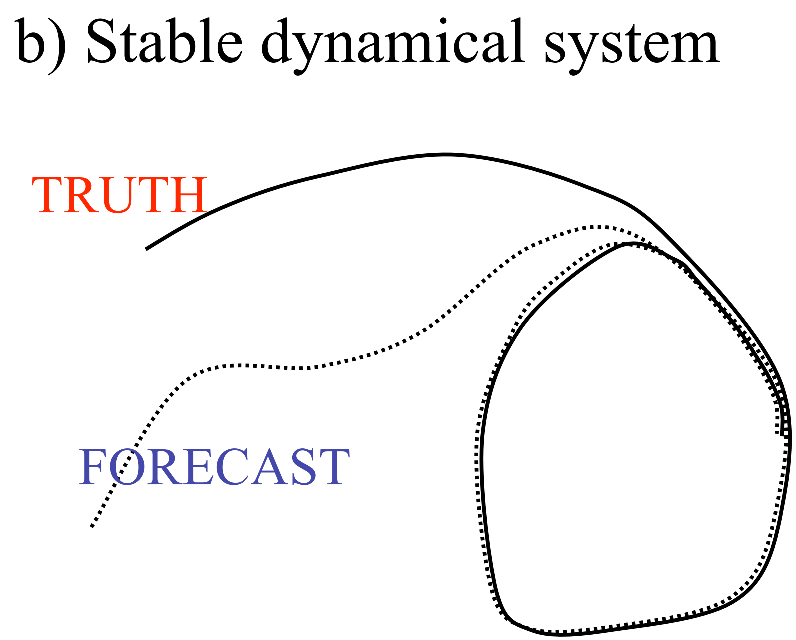
b) Stable systems are infinitely predictable

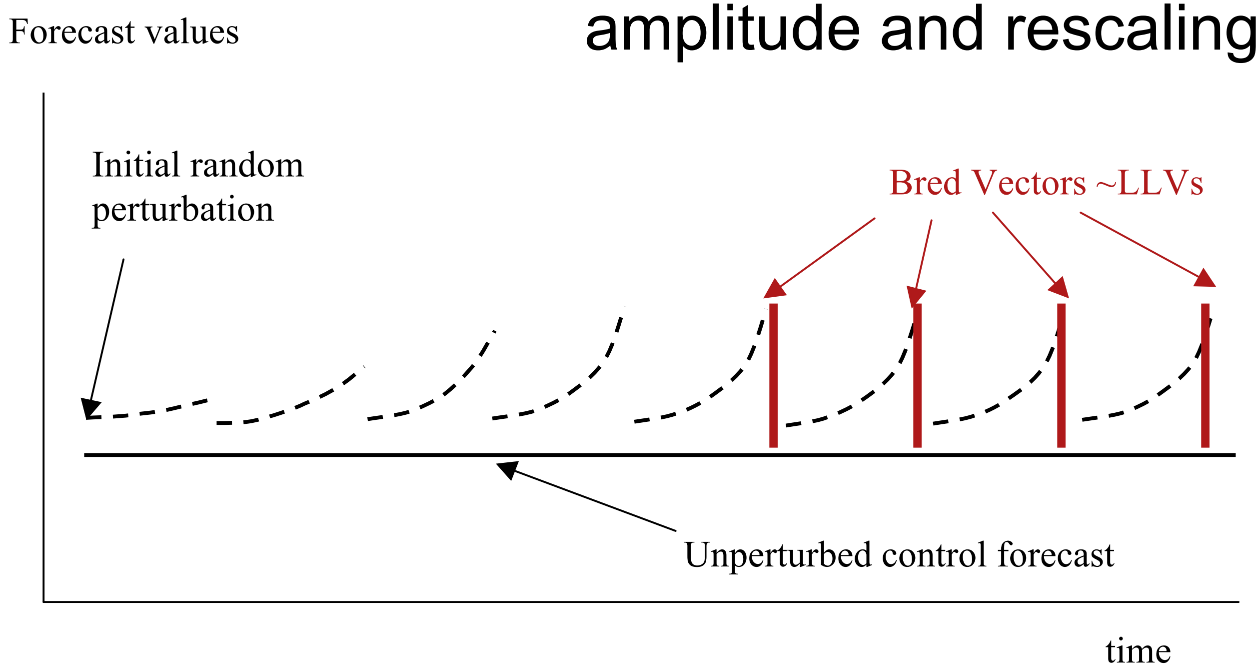




Most unstable shape: local LV ~ BV ~ makes forecast errors grow

Breeding: simply running the nonlinear model a second time, from perturbed initial conditions.

Only two tuning parameters: rescaling amplitude and rescaling interval



Local breeding growth rate: $$g(t) = \frac{1}{n\Delta t} \ln\left(|\delta \mathbf{x}| / |\delta \mathbf{x}_0| \right)$$

BVs: non linear, finite time generalization of Lyapunov vectors

A simple chaotic model:
Lorenz (1963) 3-variable model

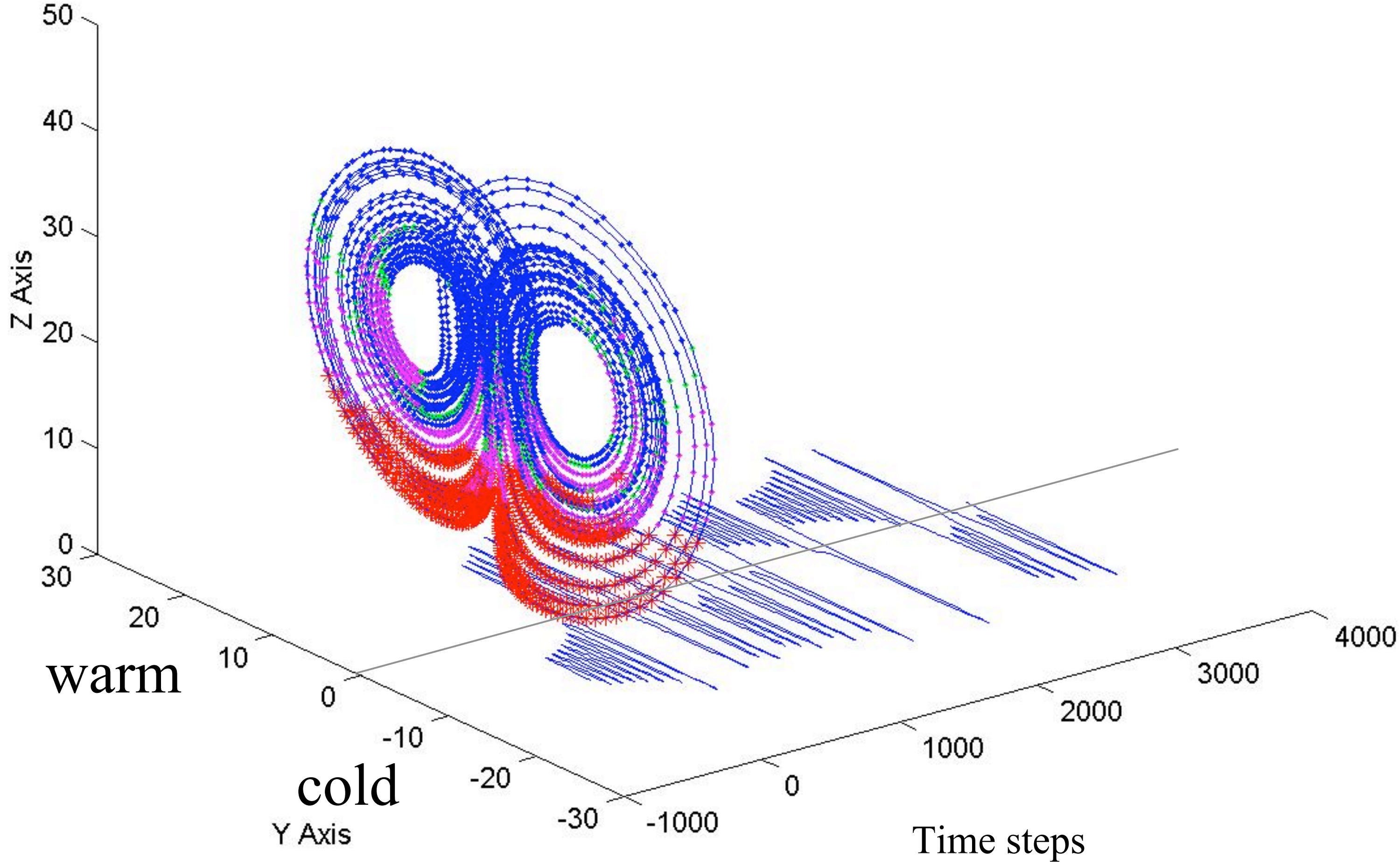
Has two regimes and the transition between them is chaotic

$$\frac{dx}{dt} = \sigma(y - x)$$

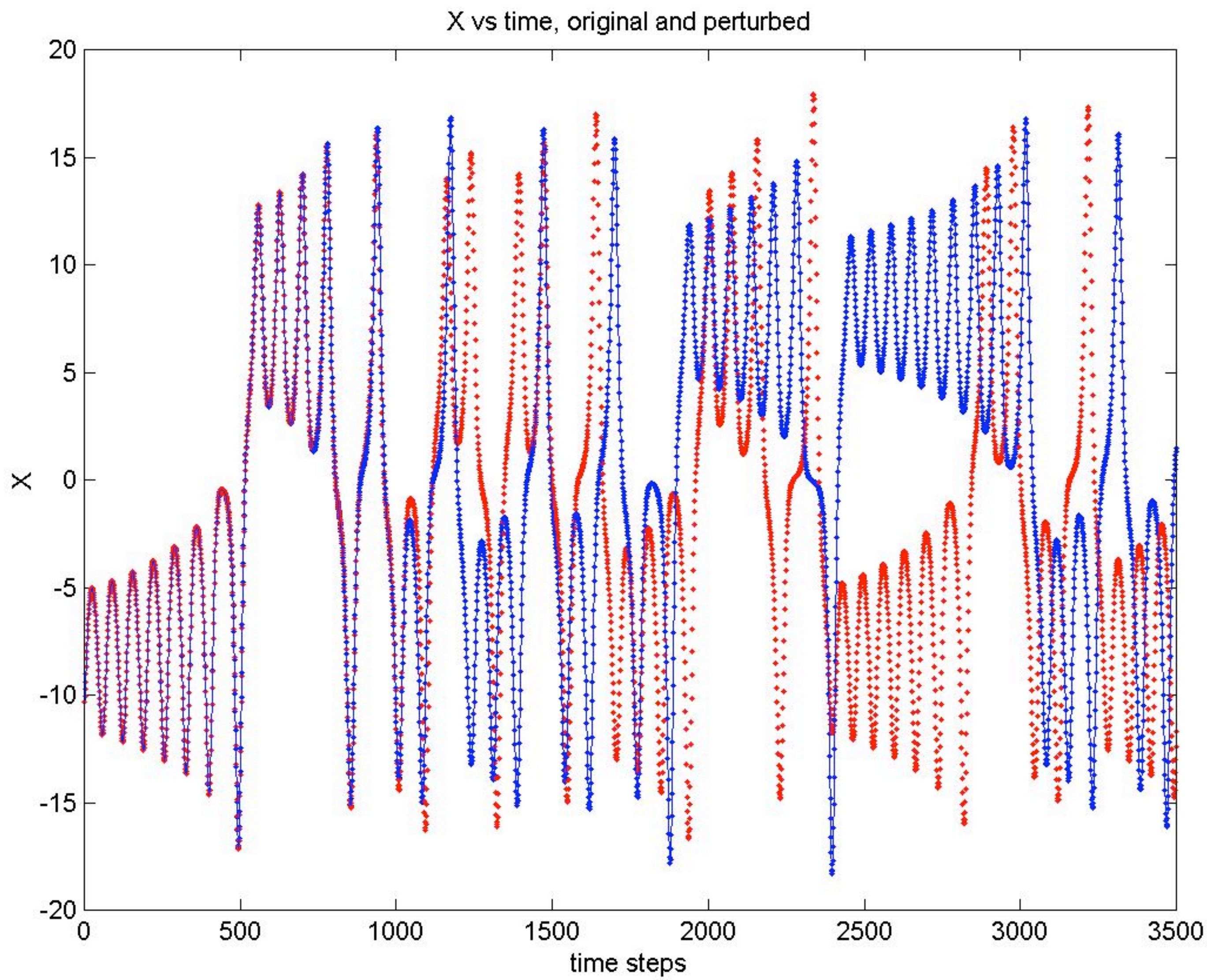
$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$

Example: Lorenz (1963) model, y(t)



Lorenz introduced an infinitesimal perturbation in the initial conditions, and the two solutions diverged!



Definition of Deterministic Chaos
(Lorenz, March 2006, 89 yrs)

WHEN THE PRESENT DETERMINES

THE FUTURE

BUT

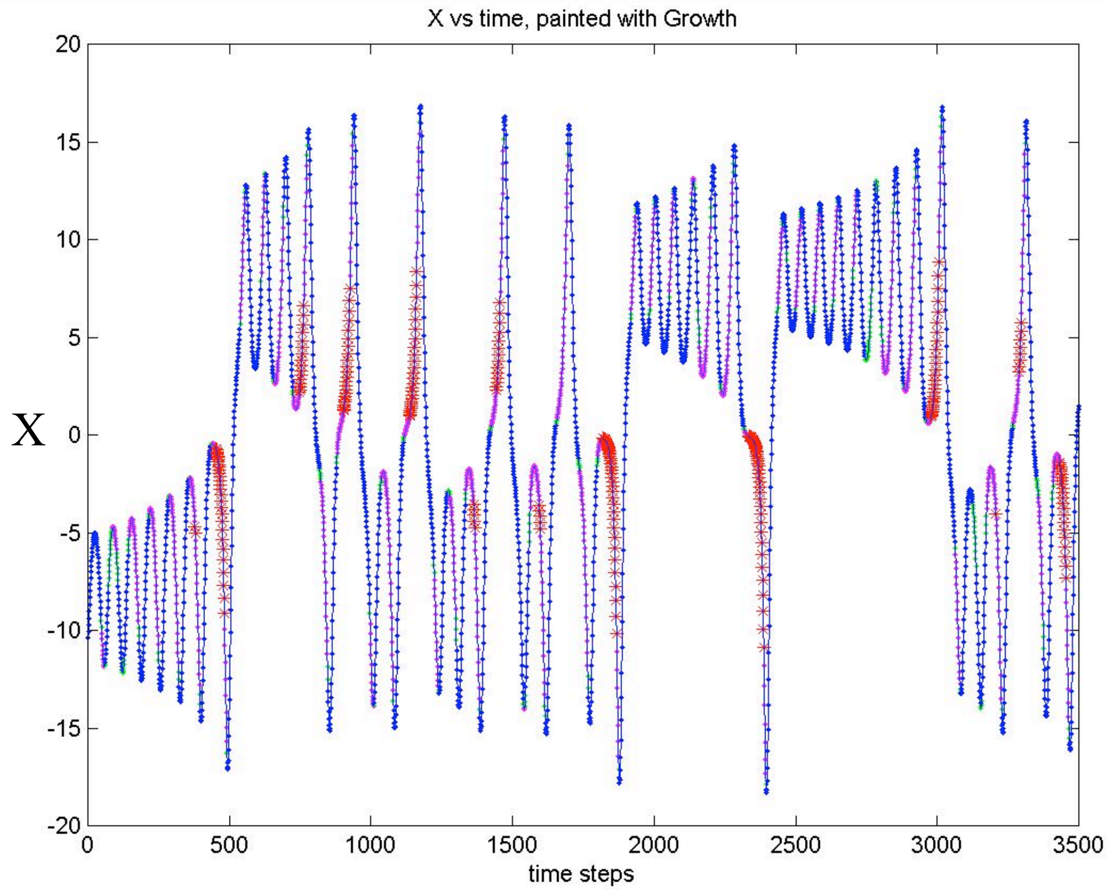
THE APPROXIMATE PRESENT DOES NOT

APPROXIMATELY DETERMINE THE FUTURE

Forecasting rules for the Lorenz model:

“warm”

“cold”



Growth rate of bred vectors:

A * indicates fast growth (>1.8 in 8 steps)

Regime change:The presence of red stars (fast BV growth) indicates that the next orbit will be the last one in the present regime.

Regime duration: One or two red stars, next regime will be short. Several red stars: the next regime will be long lasting.

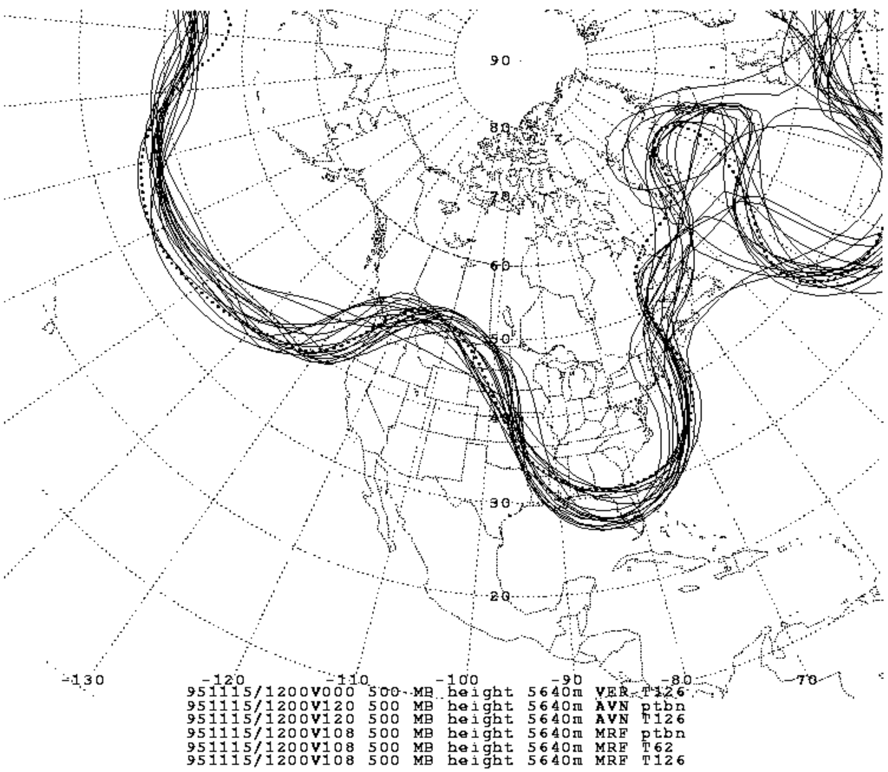
These rules surprised Lorenz himself!

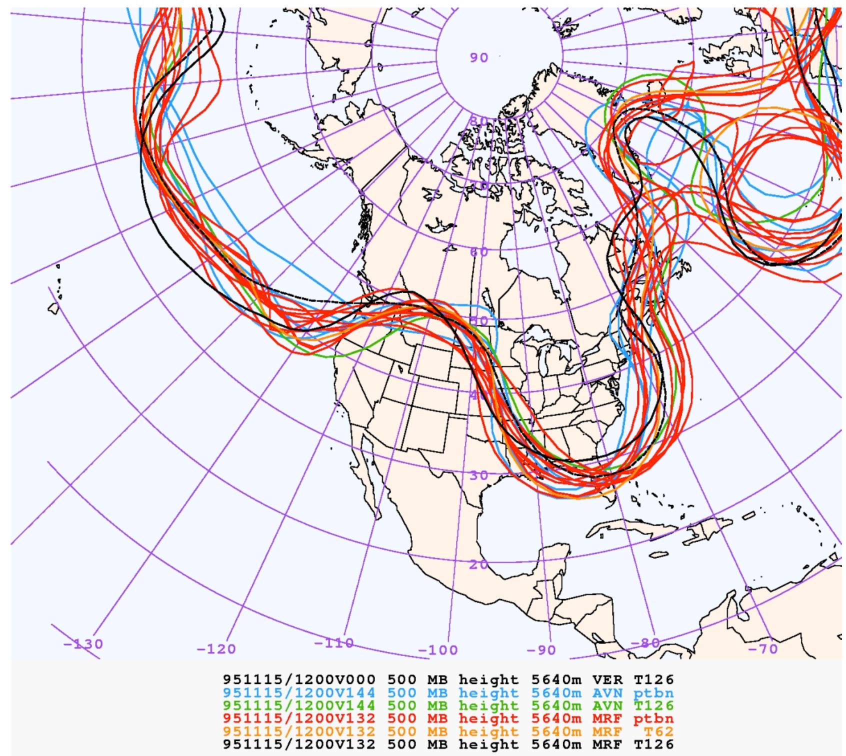
Why do breeding?

- Toth and Kalnay (1993, 1997) wanted to include in the initial conditions for ensemble forecasting the type of growing errors that would be present in the analysis
- Since all perturbations develop the shape of dominant growing errors, breeding is simple and practical
- In order to avoid collapsing into too few growing directions (LLVs), it is good to “sprinkle” the BVs with small random perturbations. This “refreshing” avoids the collapse of BVs and ensures that all unstable directions are explored

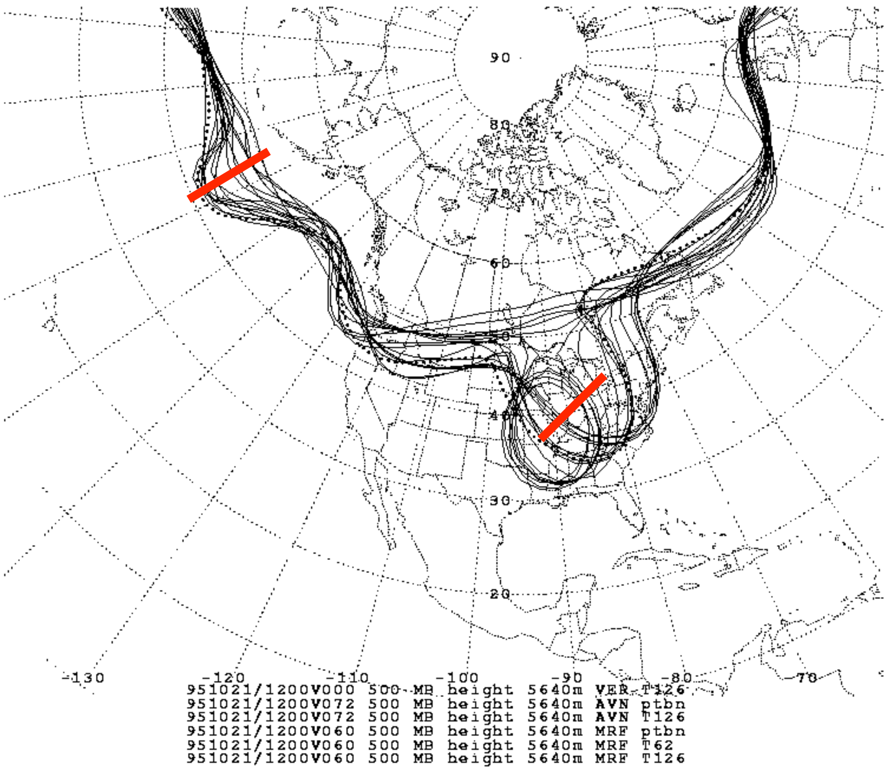
The errors of the day are instabilities of the background flow. At the same verification time, the forecast uncertainties have *the same shape*

4 days and 6 days ensemble forecasts verifying on 15 Nov 1995





Strong instabilities of the background tend to have simple shapes (perturbations lie in a low-dimensional subspace of bred vectors)



2.5 day forecast verifying on 95/10/21.

Note that the bred vectors (difference between the forecasts) lie on a 1-D space

This simplicity (**local low-dimensionality**, Patil et al. 2000) inspired the Local Ensemble Transform Kalman Filter (Ott et al. 2004, Hunt et al., 2007)

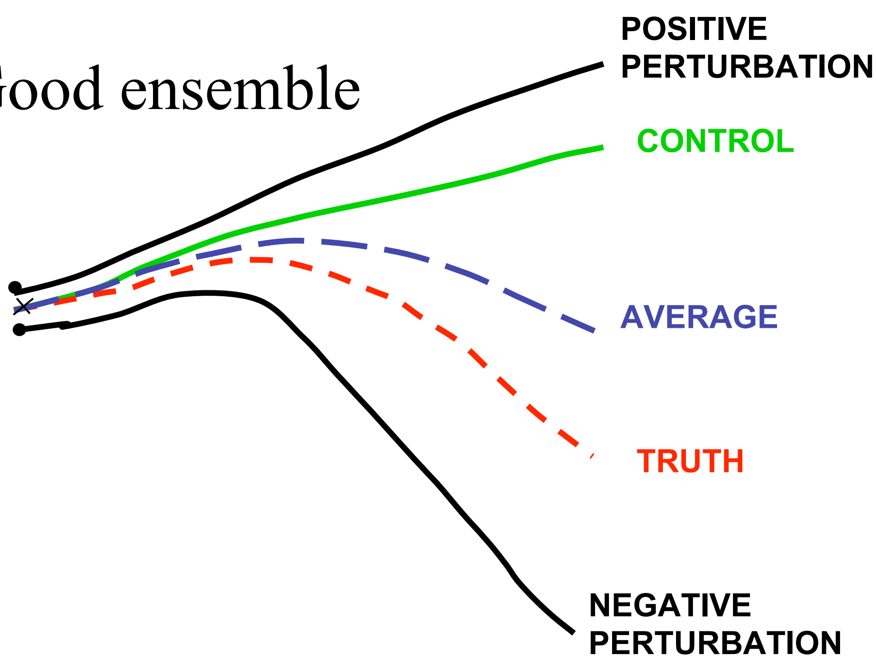
Components of ensemble forecasts

An ensemble forecast starts from initial perturbations to the analysis…

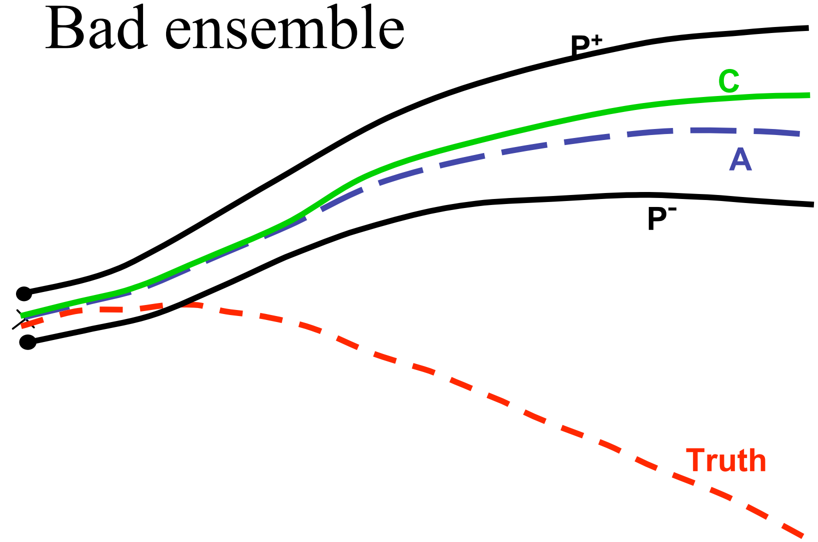
In a good ensemble “truth” looks like a member of the ensemble

The initial perturbations should reflect the analysis “errors of the day”

Good ensemble



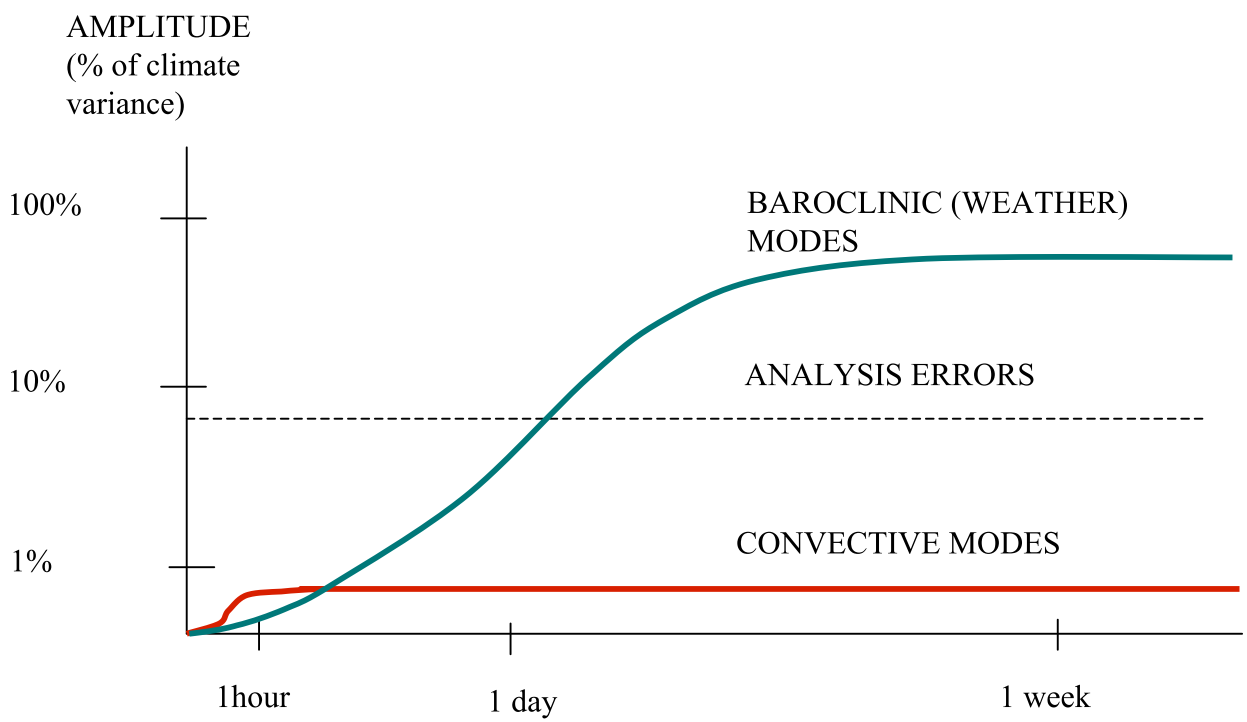
Bad ensemble



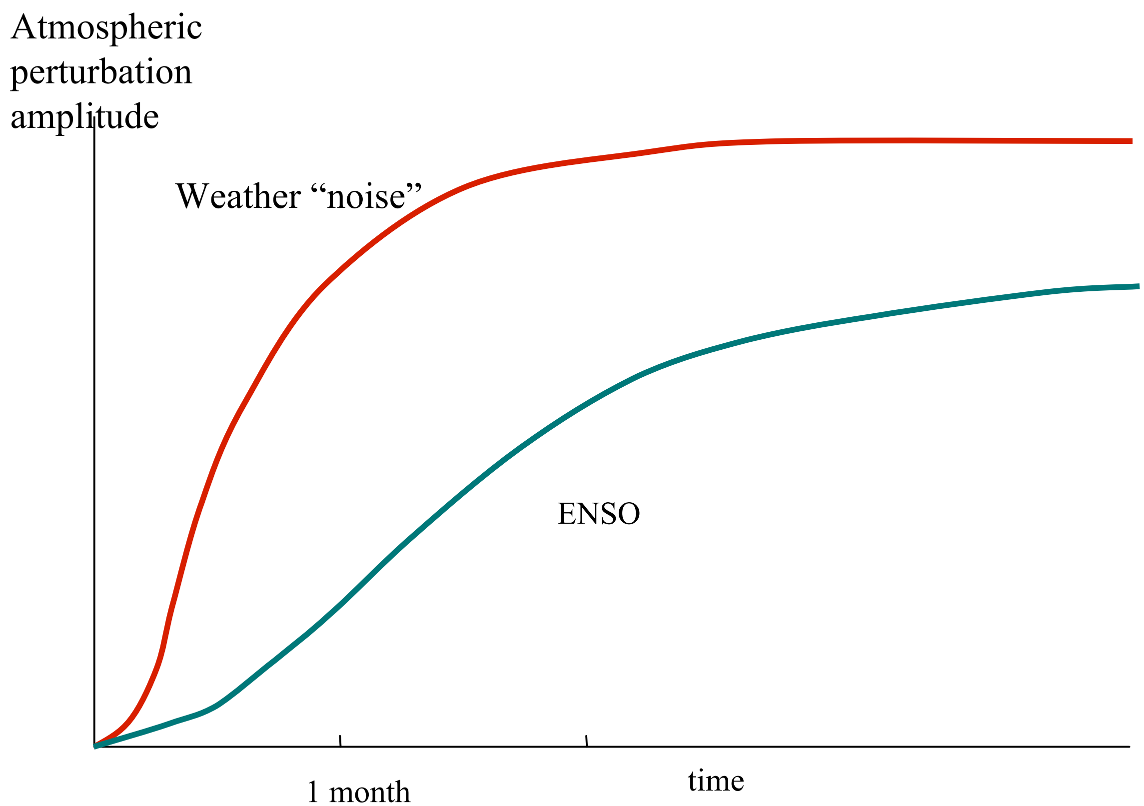
Data assimilation and ensemble forecasting in a coupled ocean-atmosphere system

- A coupled ocean-atmosphere system contains growing instabilities with many different time scales
  - The problem is to isolate the slow, coupled instability related to the ENSO variability.
- Results from breeding in the Zebiak and Cane model (Cai et al., 2002) demonstrated that
  - The dominant bred mode is the slow growing instability associated with ENSO
  - The breeding method has potential impact on ENSO forecast skill, including postponing the error growth in the “spring barrier”.
- Results from breeding in a coupled Lorenz model show that using amplitude and rescaling intervals chosen based on time scales, breeding can be used to separate slow and fast solutions in a coupled system.

Nonlinear saturation allows filtering unwanted fast, small amplitude, growing instabilities like convection (Toth and Kalnay, 1993). This is not possible with linear approaches like Lyapunov vectors and Singular Vectors.



In the case of coupled ocean-atmosphere modes, we cannot take advantage of the small amplitude of the "weather noise"! We can only use the fact that the coupled ocean modes are slower…



We coupled a slow and a fast Lorenz (1963) 3-variable model

Fast equations

$$\frac{dx_1}{dt} = \sigma(y_1 - x_1) - C_1(Sx_2 + O)$$

$$\frac{dy_1}{dt} = rx_1 - y_1 - x_1 z_1 + C_1(Sy_2 + O)$$

$$\frac{dz_1}{dt} = x_1 y_1 - bz_1 + C_1(Sz_2)$$

Slow equations

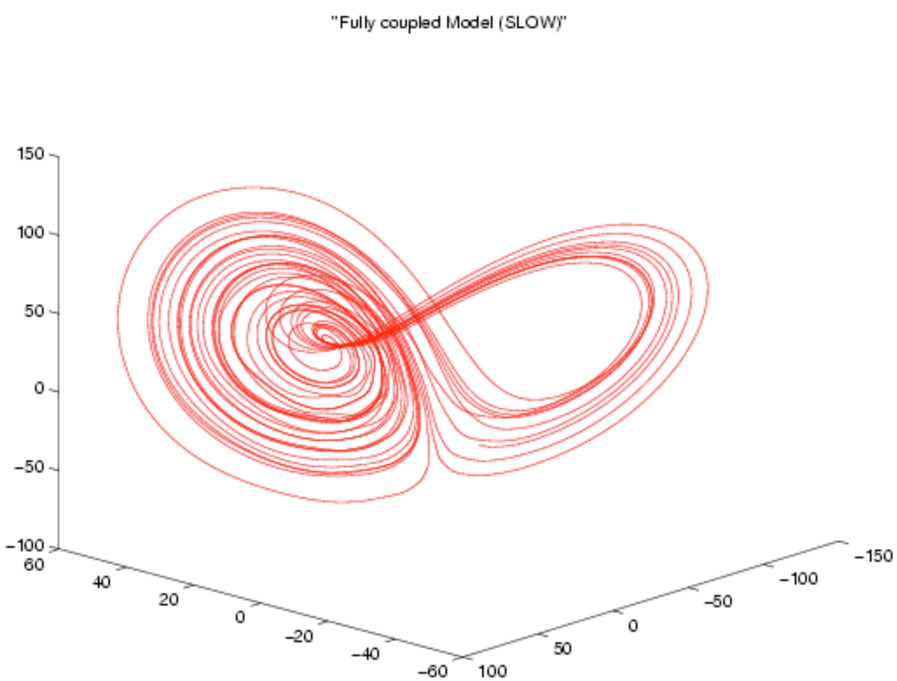
$$\frac{1}{\tau}\frac{dx_2}{dt} = \sigma(y_2 - x_2) - C_2(x_1 + O)$$

$$\frac{1}{\tau}\frac{dy_2}{dt} = rx_2 - y_2 - Sx_2 z_2 + C_2(y_1 + O)$$

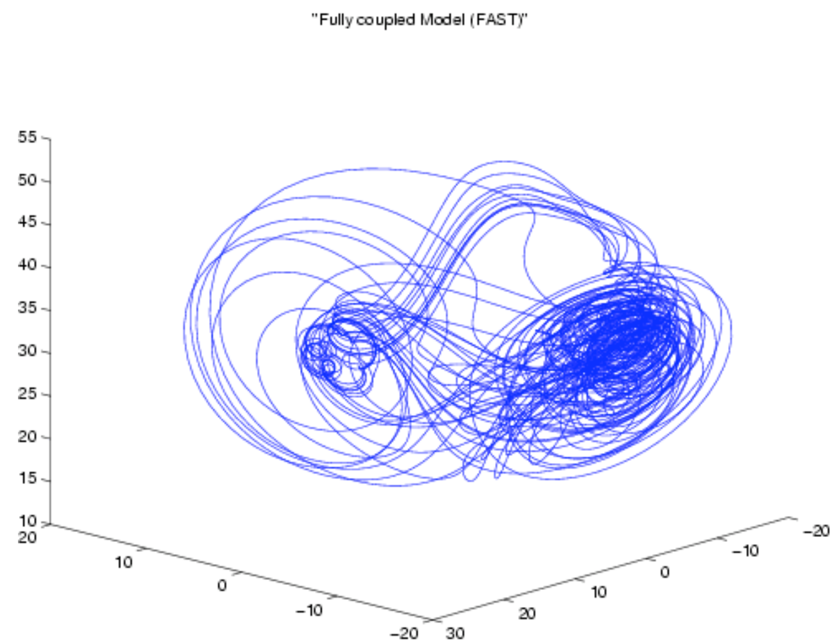
$$\frac{1}{\tau}\frac{dz_2}{dt} = Sx_2 y_2 - bz_2 + C_2(z_1)$$

Now we test the fully coupled "ENSO-like" system,
with similar amplitudes between "slow signal" and "fast noise"

"slow ocean"

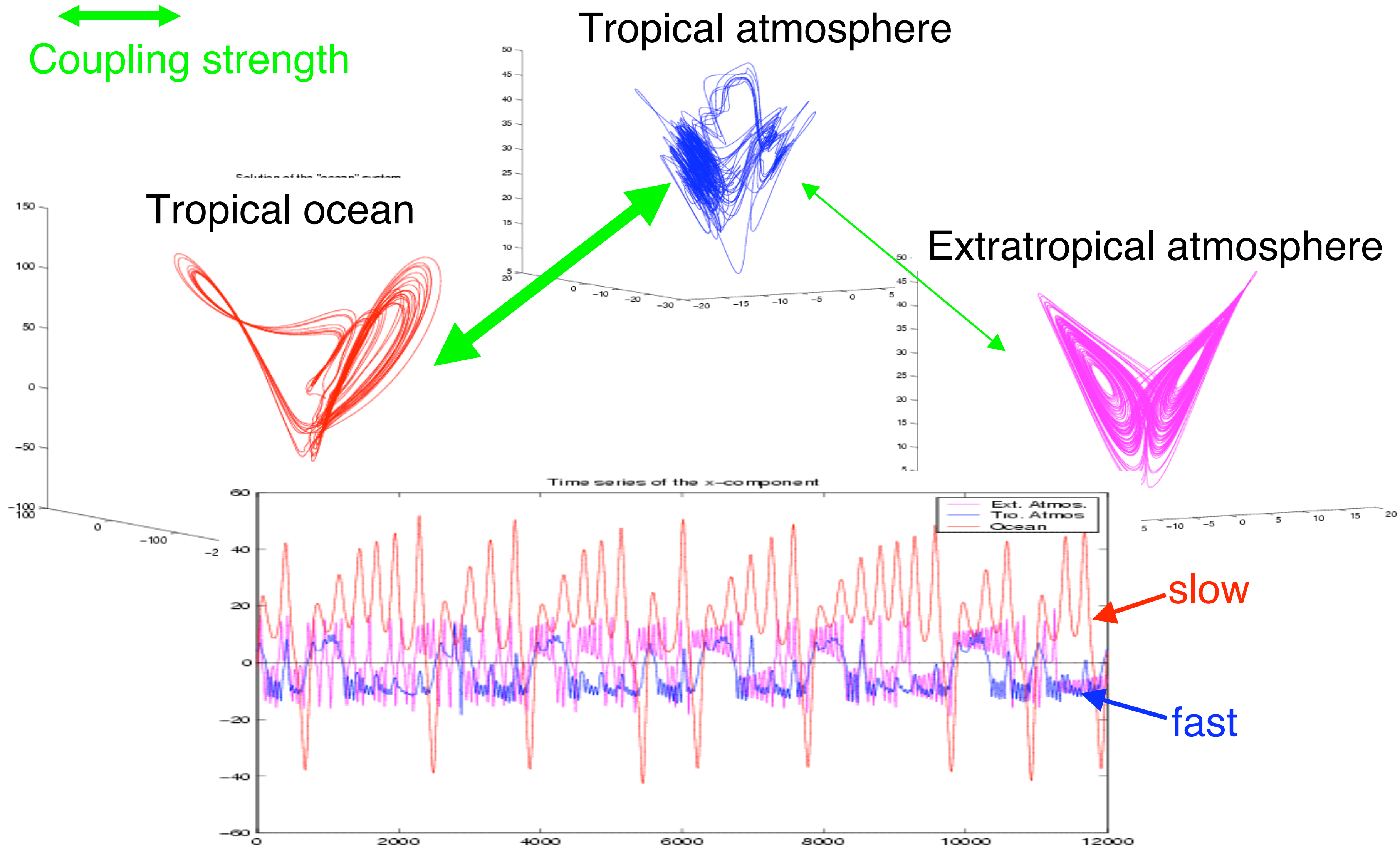


"tropical atmosphere"

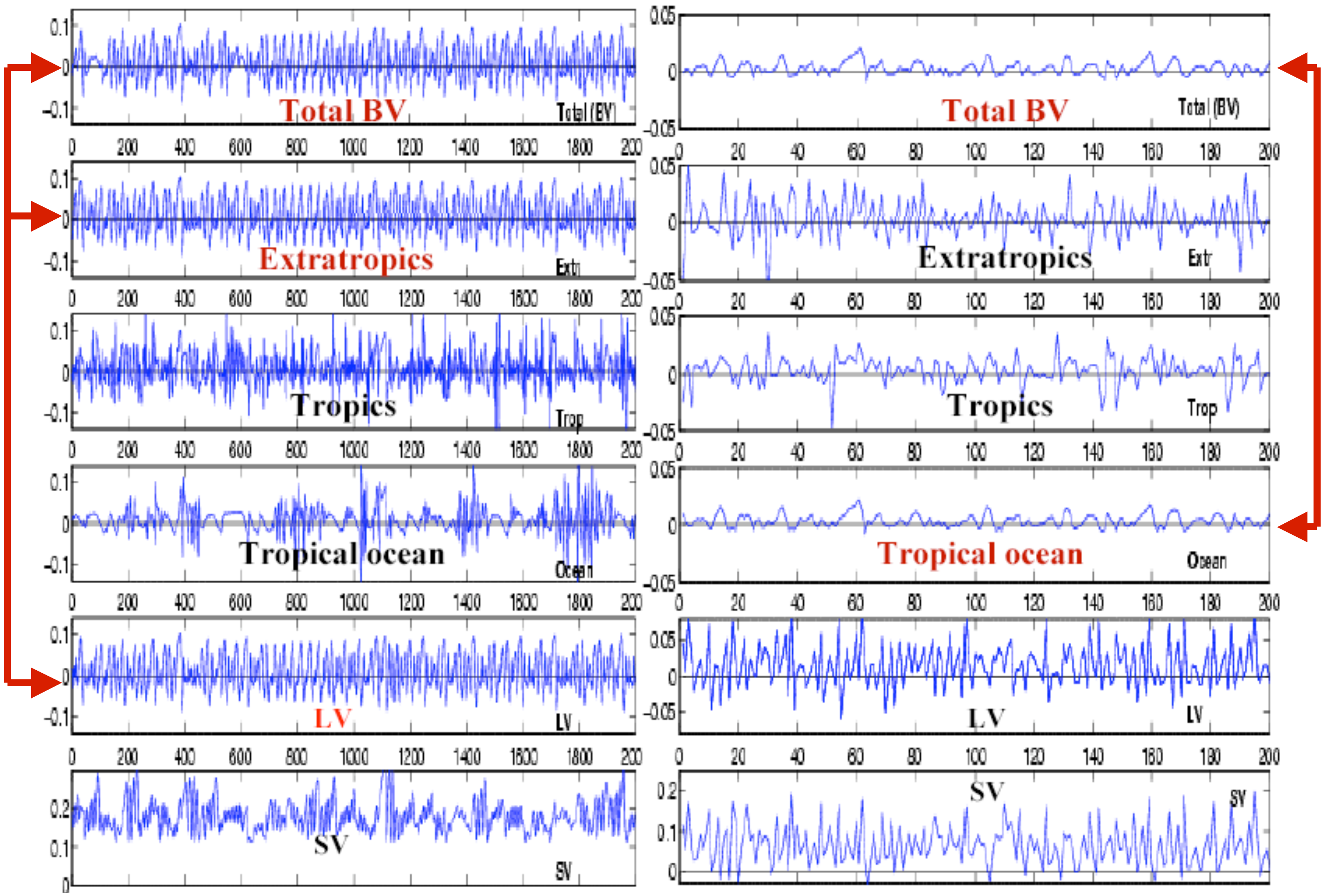


Then we added an extratropical atmosphere coupled with the tropics

Coupled fast and slow Lorenz 3-variable models (Peña and Kalnay, 2004)



Breeding in a coupled Lorenz model



Short rescaling interval (5 steps) and small amplitude: fast modes

Long rescaling interval (50 steps) and large amplitude: ENSO modes

The linear approaches (LV, SV) cannot capture the slow ENSO signal

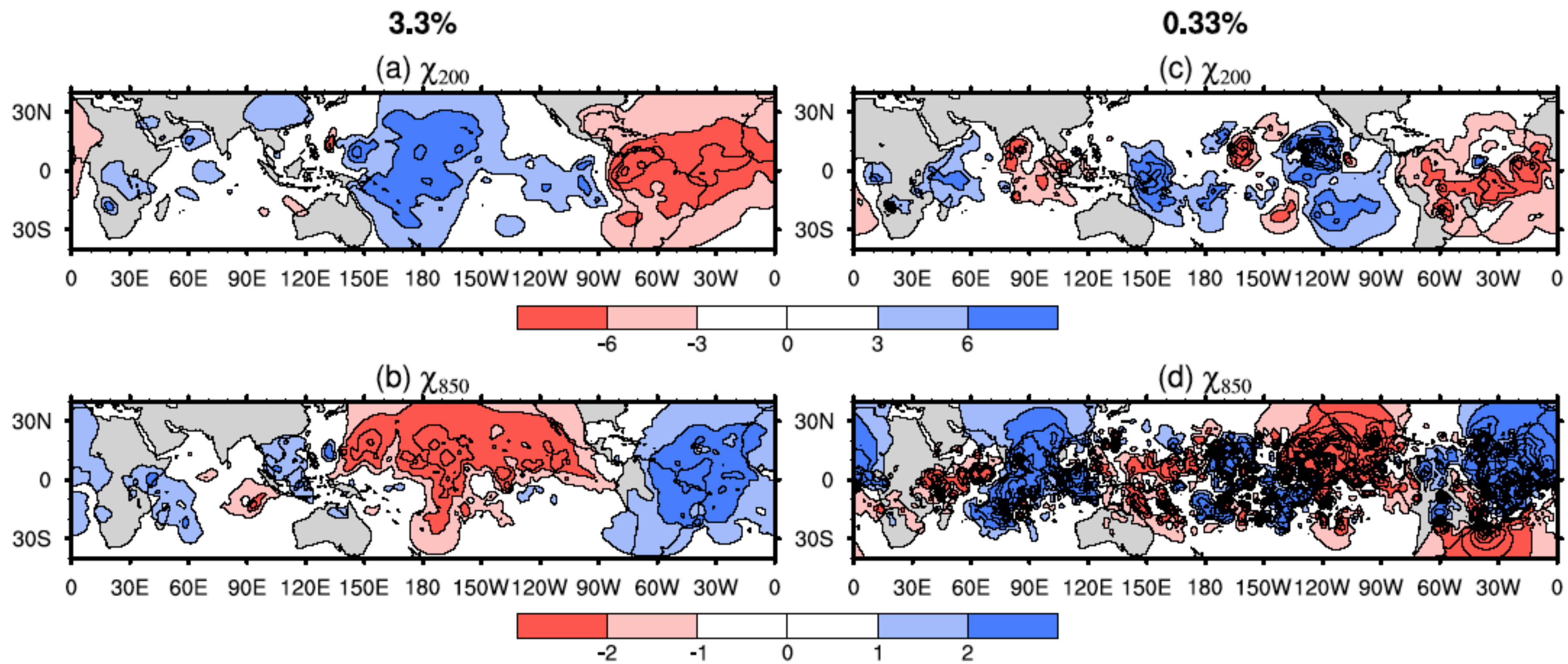
From Lorenz coupled models:

- In coupled fast/slow models, we can do breeding to isolate the slow modes
- We have to choose a slow variable and a long interval for the rescaling
- This is true for nonlinear approaches (e.g., EnKF) but not for linear approaches (e.g., SVs, LVs)
- This has been applied to ENSO coupled instabilities:
  - Cane-Zebiak model (Cai et al, 2003)
  - **NASA and NCEP fully coupled GCMs (Yang et al, 2006)**
  - **NASA operational system with real observations** (Yang et al. 2008)

Examples of breeding in a coupled ocean-atmosphere system with coupled instabilities

- In coupled fast/slow models, we can do breeding to isolate the slow modes
- We have to choose a slow variable and a long interval for the rescaling
- This identifies coupled instabilities.
- Examples
  - Madden-Julian Bred Vectors
  - NASA operational system with real observations (Yang et al 2007, MWR)
  - Ocean instabilities and their physical mechanisms (Hoffman et al, 2008, with thanks to Istvan Szunyogh)

Chikamoto et al (2007, GRL): They found the Madden-Julian instabilities BV by choosing an appropriate rescaling amplitude (only within the tropics)

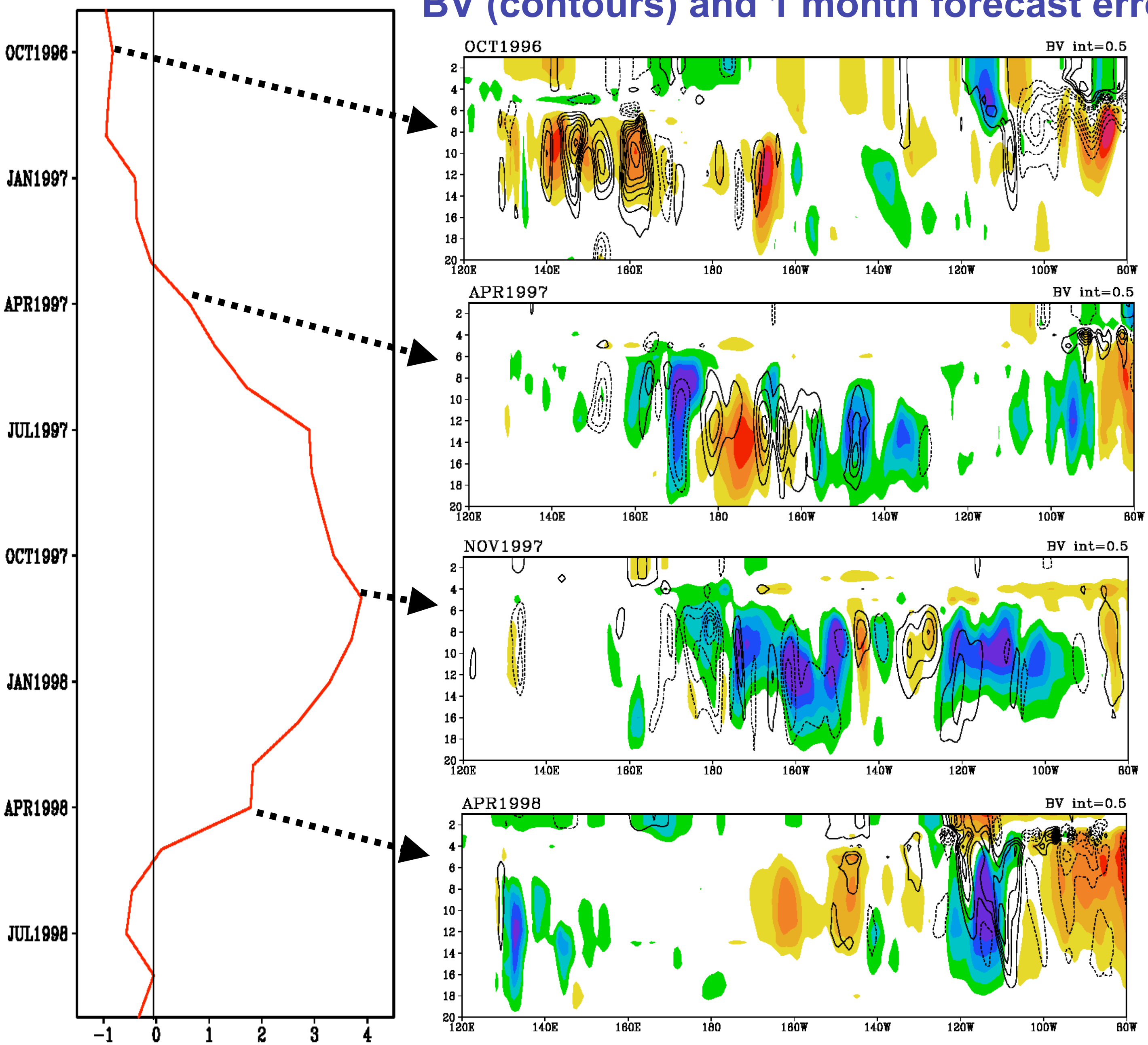


Finding the shape of the errors in El Niño forecasts to improve data assimilation

- **Bred vectors**:
  - Differences between the control forecast and perturbed runs:
  - **Should show the shape of growing errors (?)**
- **Advantages**
  - Low computational cost (two runs)
  - Capture coupled instabilities
  - Improve data assimilation

Niño3 index

Yang (2005): Vertical cross-section at Equator for BV (contours) and 1 month forecast error (color)



Before 97' El Niño, error is located in W. Pacific and near coast region

During development, error shifts to lower levels of C. Pacific.

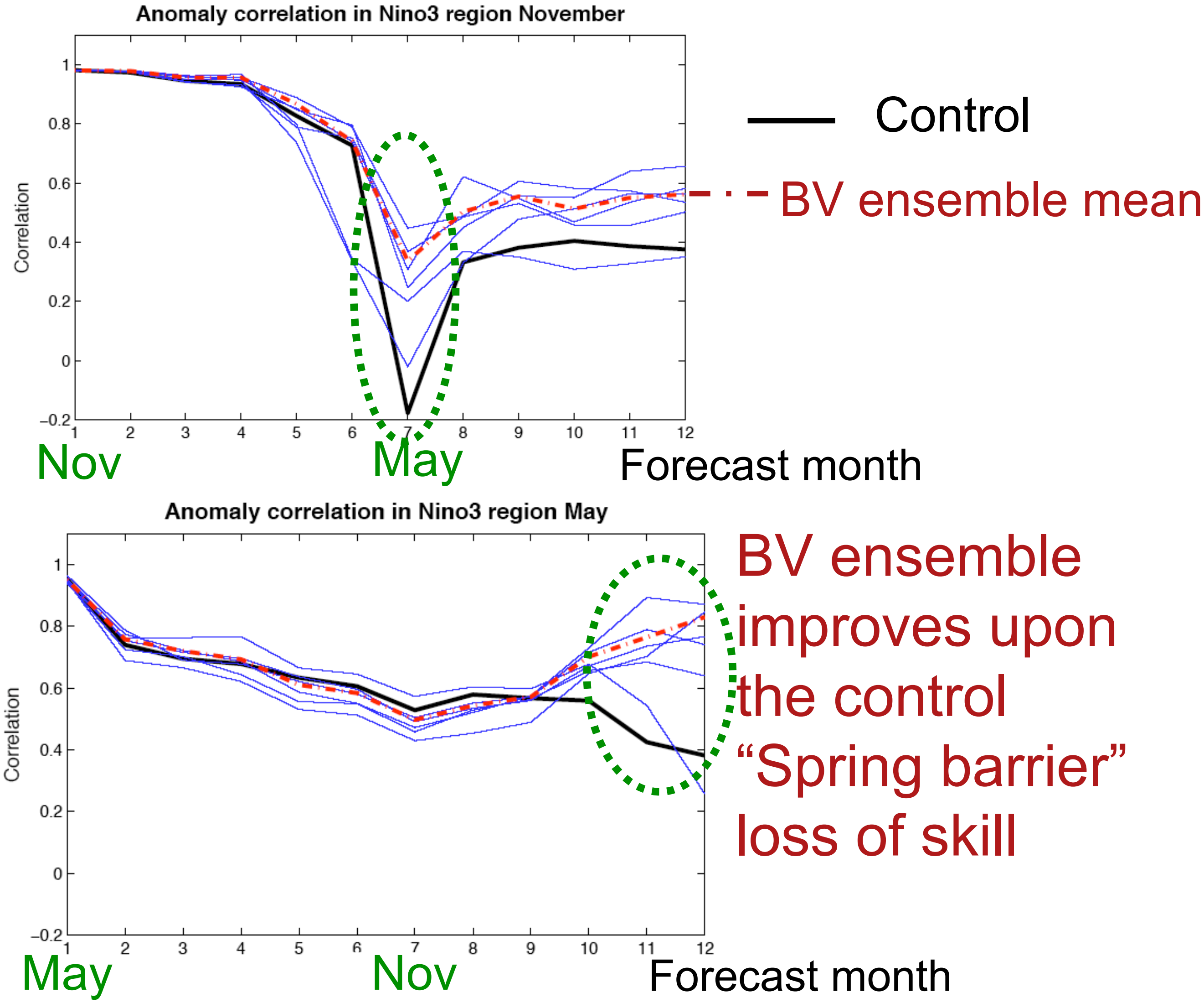
At mature stage, error shifts further east and it is smallest near the coast.

After the event, error is located mostly in E. Pacific.

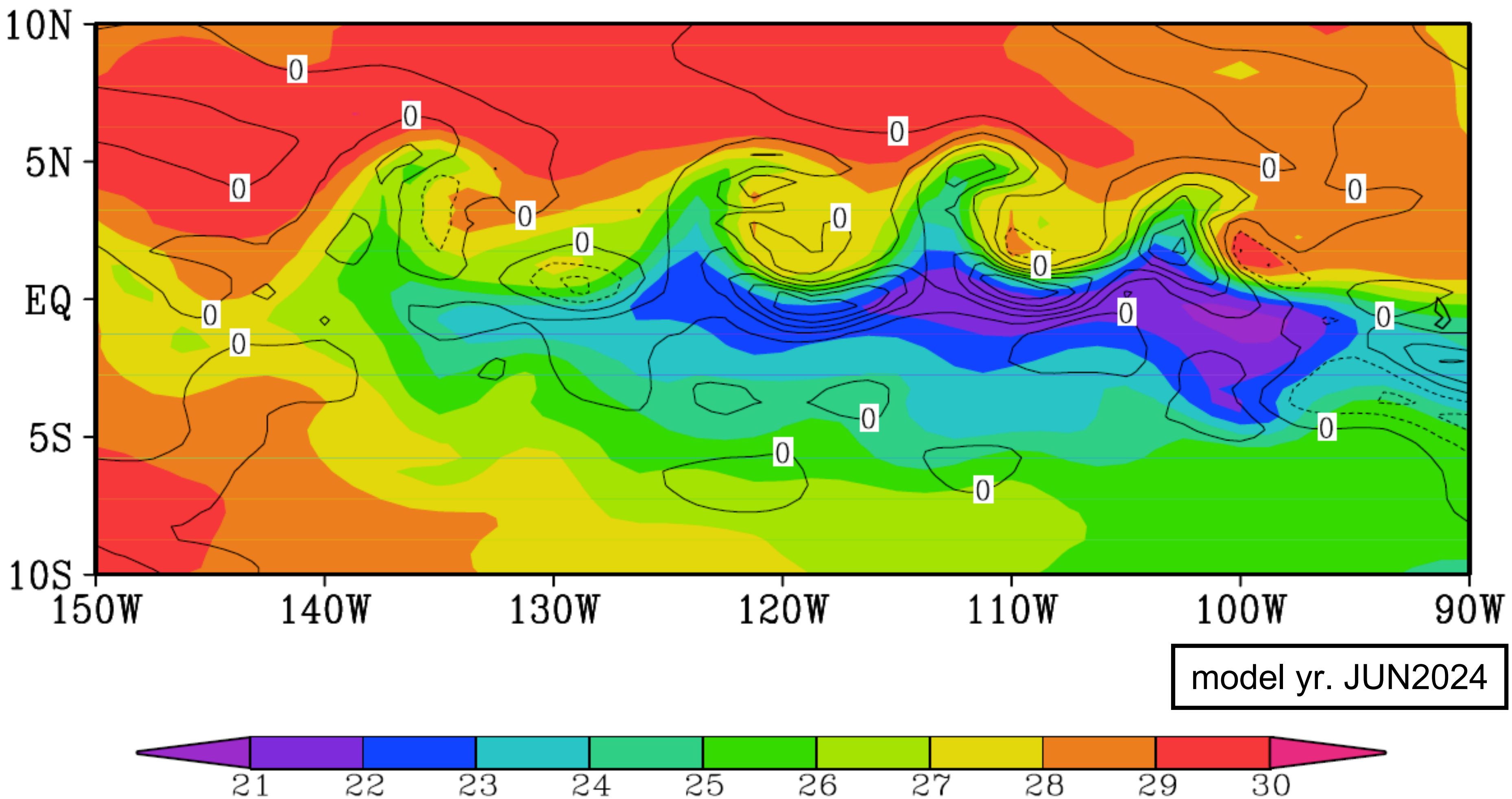
Yang: Impact of forecasts of El Niño with 3 pairs of BVs: November and May restarts (1993-2002)

Start from cold season

Start from warm season

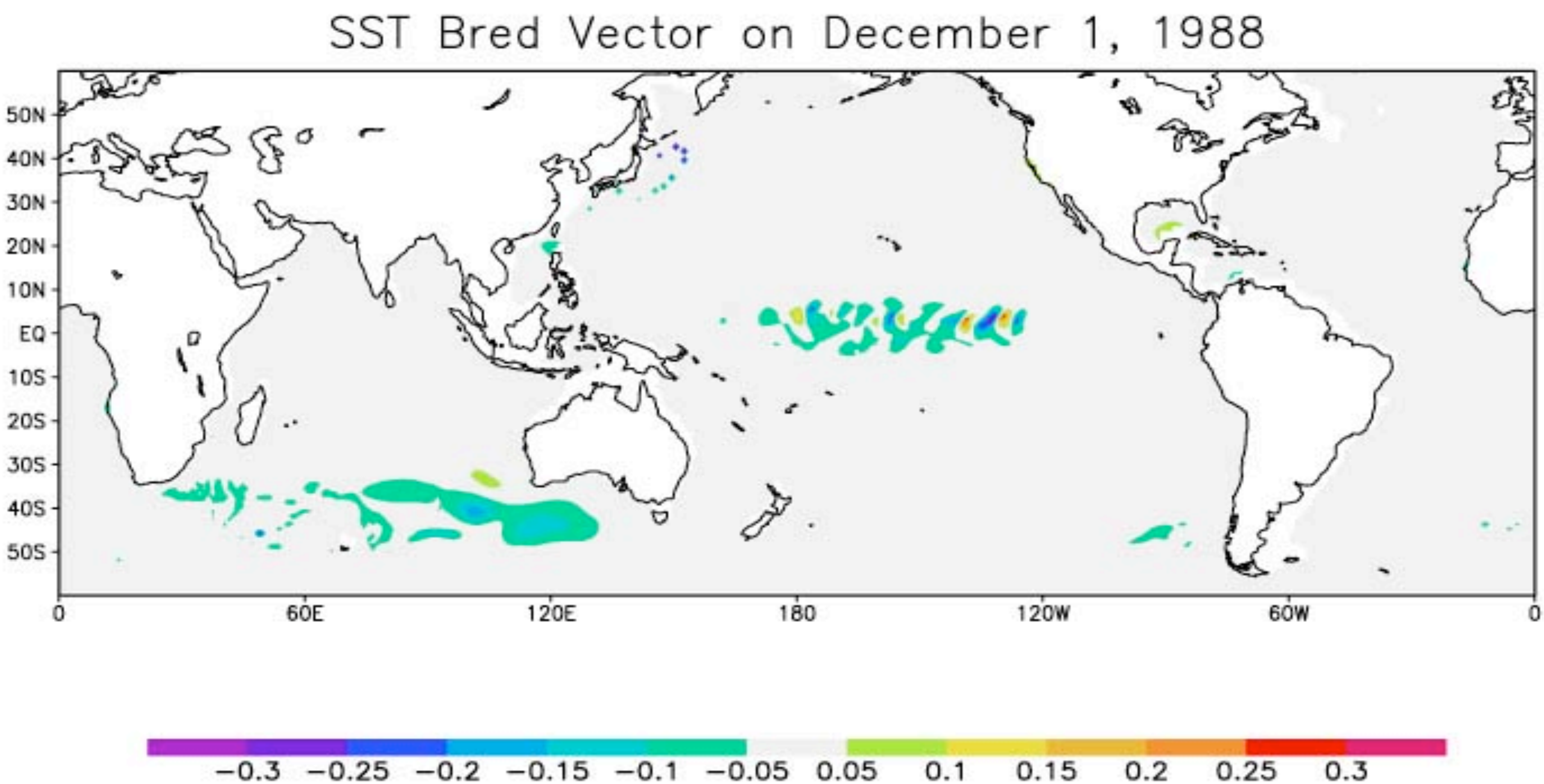


Yang et al., 2006: Bred Vectors (contours) overlay Tropical Instability waves (SST): making them grow and break!

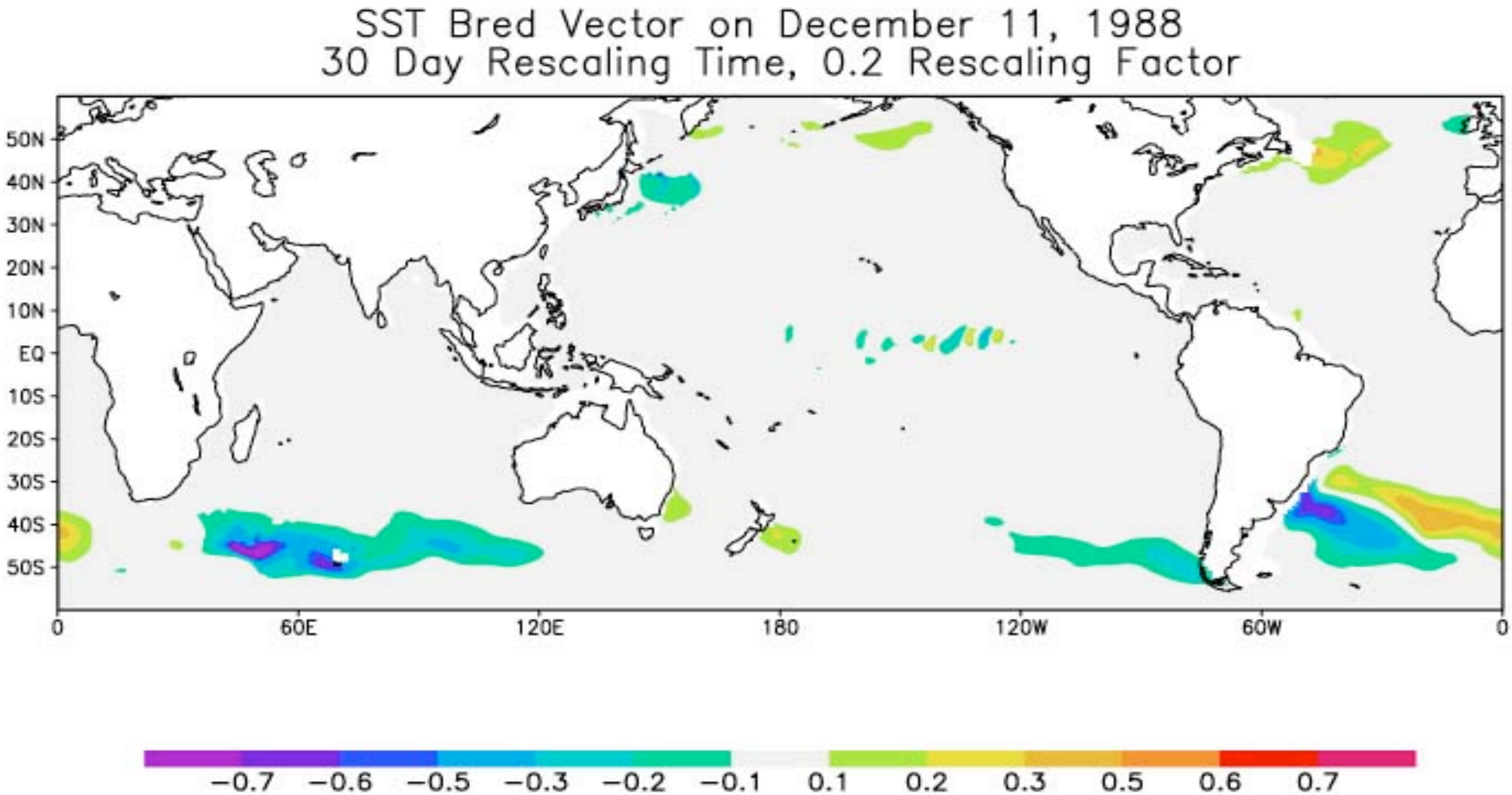


Hoffman et al (2008): finding ocean instabilities with breeding time-scale 10-days captures tropical instabilities

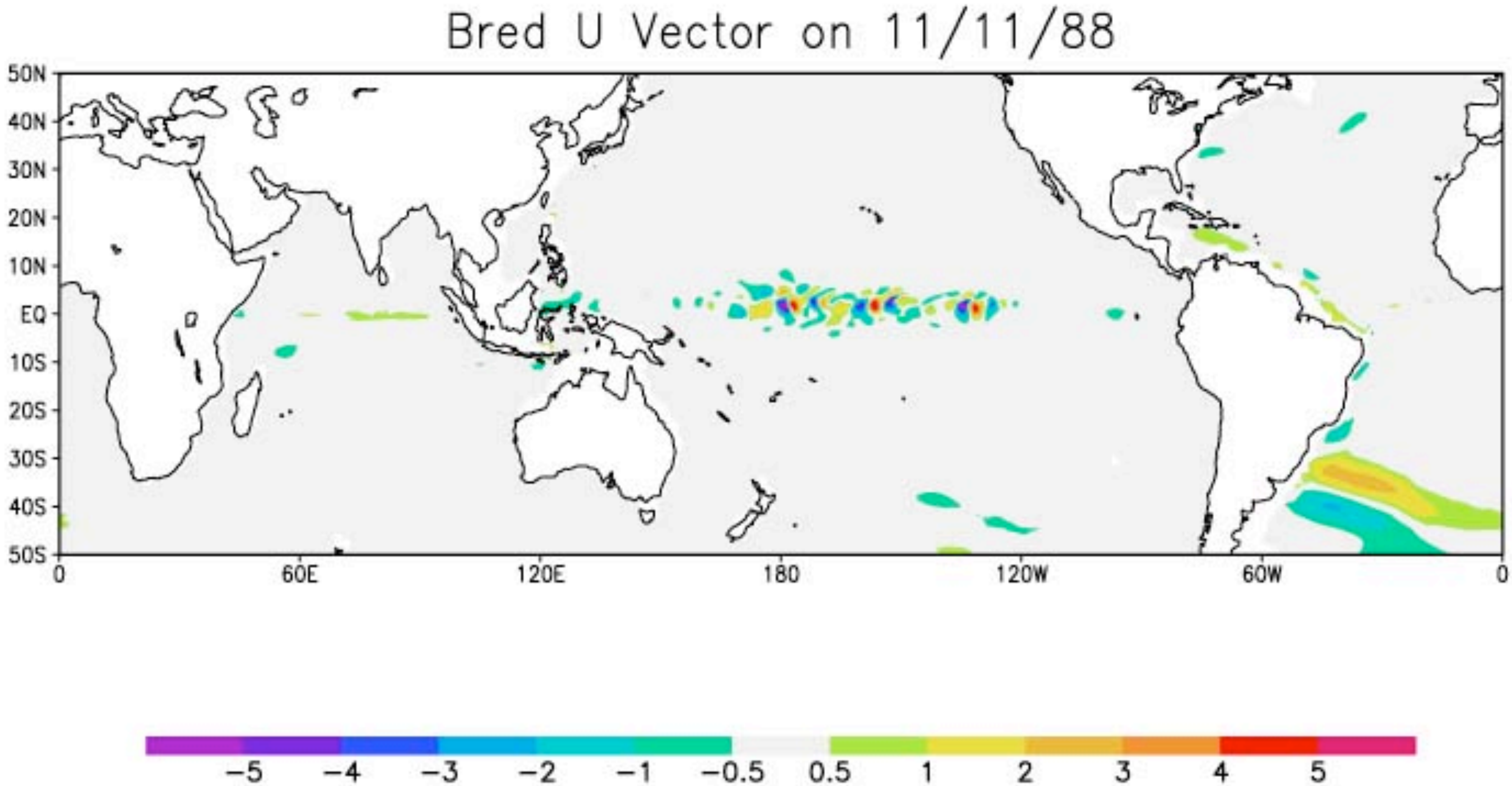
Breeding time scale: 10 days



When the rescaling time scale is 30 days, extratropical instabilities dominate



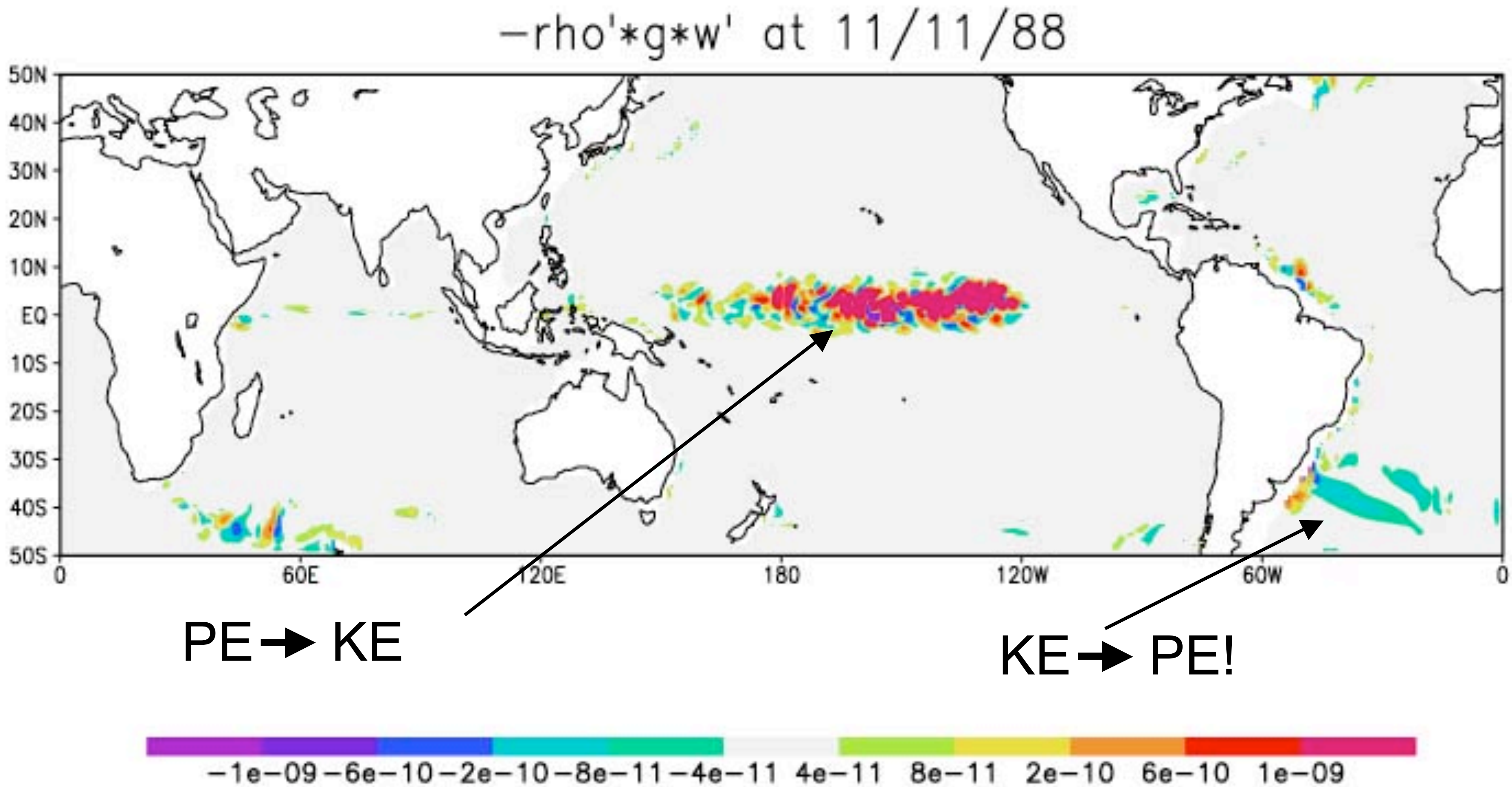
Here we have both tropical and “South Atlantic Convergence Zone” instabilities. Can we determine the dynamic origin of the instabilities?



The Bred Vector Kinetic Energy equation can be computed exactly because both control solution and perturbed solution satisfy the full equations!

$$\frac{\partial KE_{bv}}{\partial t} = horizontal\ \ fluxes - \rho_b g w_b + \ldots$$

Conversion from potential to kinetic energy



Summary: We can fight chaos and extend predictability by understanding error growth

- Chaos is not random: it is generated by physical instabilities
- Breeding is a simple and powerful method to find the growth and shape of the instabilities
- These instabilities also dominate the forecast errors: we can use their shape to improve data assimilation.
- Ensemble Kalman Filter is the ultimate method to explore and “beat chaos” through data assimilation.
- In the “chaotic” Lorenz model the growth of bred vectors predicts regime changes and how long they will last.
- Nonlinear methods, like Breeding and EnKF, can take advantage of the saturation of fast weather noise and isolate slower instabilities.
- Bred Vectors predict well the evolution of coupled forecast errors
- Bred Vectors help explain the physical origin of ocean instabilities
- Ensembles of BV improve the seasonal and interannual forecast skill, especially during the “spring barrier”

REFERENCES: www.weatherchaos.umd.edu
www.atmos.umd.edu/~ekalnay