Comparing **Local Ensemble Transform Kalman Filter** with **4D-Var** in a Quasi-geostrophic model

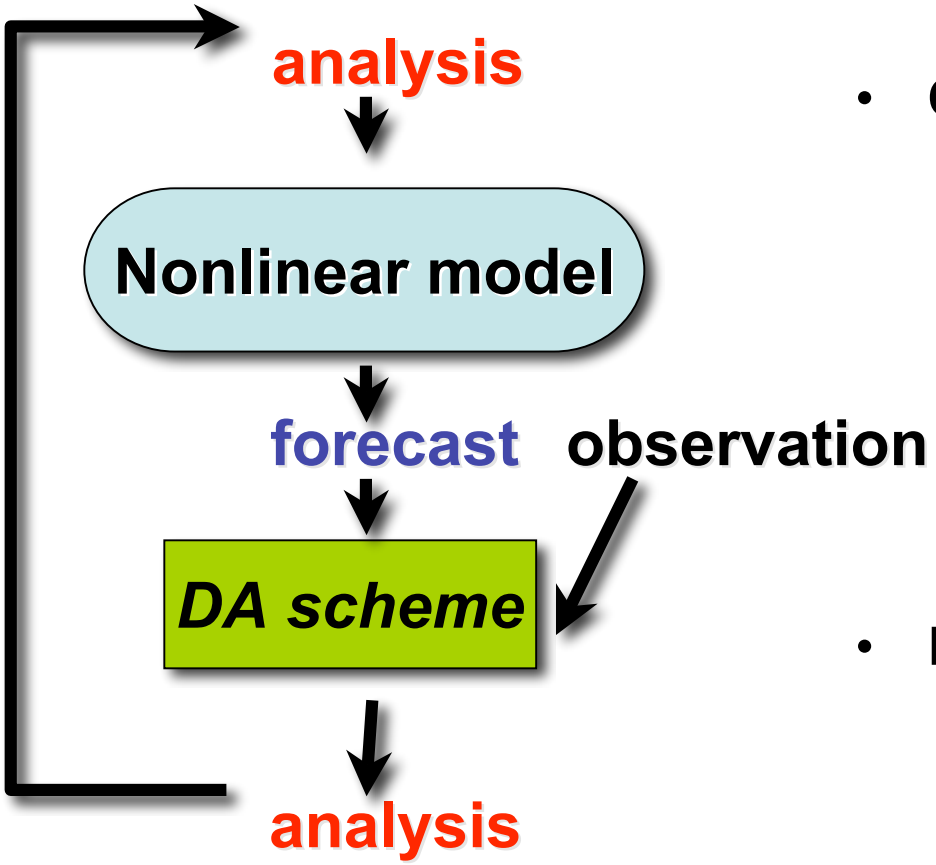
Shu-Chih Yang[1,2], Eugenia Kalnay[1],
Matteo Corazza[3], Alberto Carrassi[4] and
Takemasa Miyoshi[5]

[1] University of Maryland
[2] GMAO/NASA GFSC
[3] ARPAL CFMI-PC
[4] University of Ferrara
[5] Japan Meteorological Agency

Background

- Ensemble Kalman Filter (eg. LETKF) and 4D-Var are DA methods which can take into account the “flow-dependent errors”.
- The implementation of LETKF and 4D-Var are very different:
  - LETKF: treat model as a black box, *local*
  - 4D-Var: model dependent, *global*
- Compare the performance of LETKF and 4D-Var

Experiment setup



- **Quasi-Geostrophic Model**
  (Rotunno and Bao, 1996;Morss,1999)
  - Channel model, periodic in x
  - Horizontal: 64x33, Vertical: 7 levels
  - Model variables
    potential vorticity (q) arranged at interior 5 levels, potential temperature (θ) at top and bottom levels
- Experiment setup
  - 3% observation coverage (64 obs.)
    simulated rawinsonde (u,v,t) at all 7 levels, every 12hour
  - Analysis cycle: 12 hours
  - Initial condition, 3D-Var analysis solution

Data assimilation schemes

- 3D-Var (Morss, 1999)
  - $B_{3D\text{-}Var}$ has been optimized and is time-independent
  - Observation error covariance, **R**, is diagonal: uncorrelated between observations and between variables
  - Used as the benchmark

- Ensemble-based hybrid scheme (Corazza et al., 2002, Yang et al. 2006)
  - $B_{3D\text{-}Var}$ is augmented by the a set of bred vectors (the flow dependent errors)

    $B_{HYBD}=(1-\alpha)\, B_{3D\text{-}Var}+\alpha\, EE^{T}$. α is the hybrid coefficient

    $$[\mathbf{I}+((1-\alpha)\mathbf{B}_{3DVAR}+\alpha\mathbf{E}\mathbf{E}^{\mathbf{T}})\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}](x_a-x_b)=[(1-\alpha)\mathbf{B}_{3DVAR}+\alpha\mathbf{E}\mathbf{E}^{\mathbf{T}}]\mathbf{H}^{T}\mathbf{R}^{-1}(y-\mathbf{H}x_b)$$

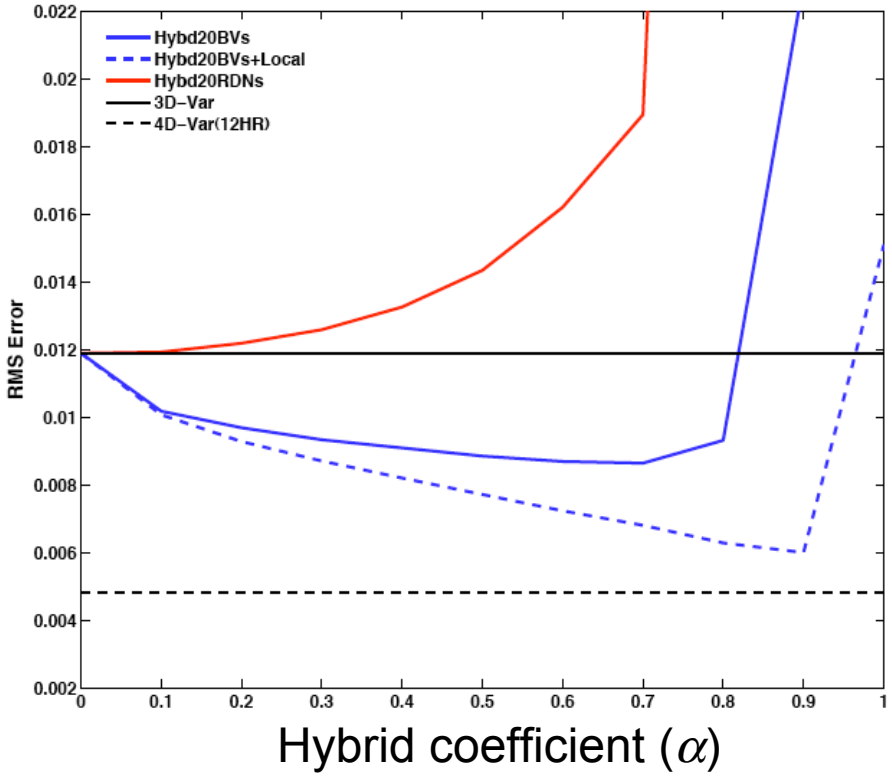
  - Implemented in the 3D-Var framework

Data assimilation schemes

- LETKF (Hunt et al., 2006)
  - An efficient method to implement Ensemble Kalman Filter
    - Perform in a local volume (19x19x7)
    - Compute matrix inverse in the space spanned by ensemble (ensemble size =40)
    - A random perturbations (3% vectors amplitude) is added to the ensemble vectors
- 4D-Var
  - The adjoint model is generated by TAMC, but need to correct several subtle bugs related to boundary conditions
  - B_0 needs to be optimized.

    $\mathbf{B}=0.02\times \mathbf{B}_{3DVAR}$

Ensemble-based hybrid scheme vs. Variational-based scheme



3D-Var

HYBD, 20 Rdn vectors

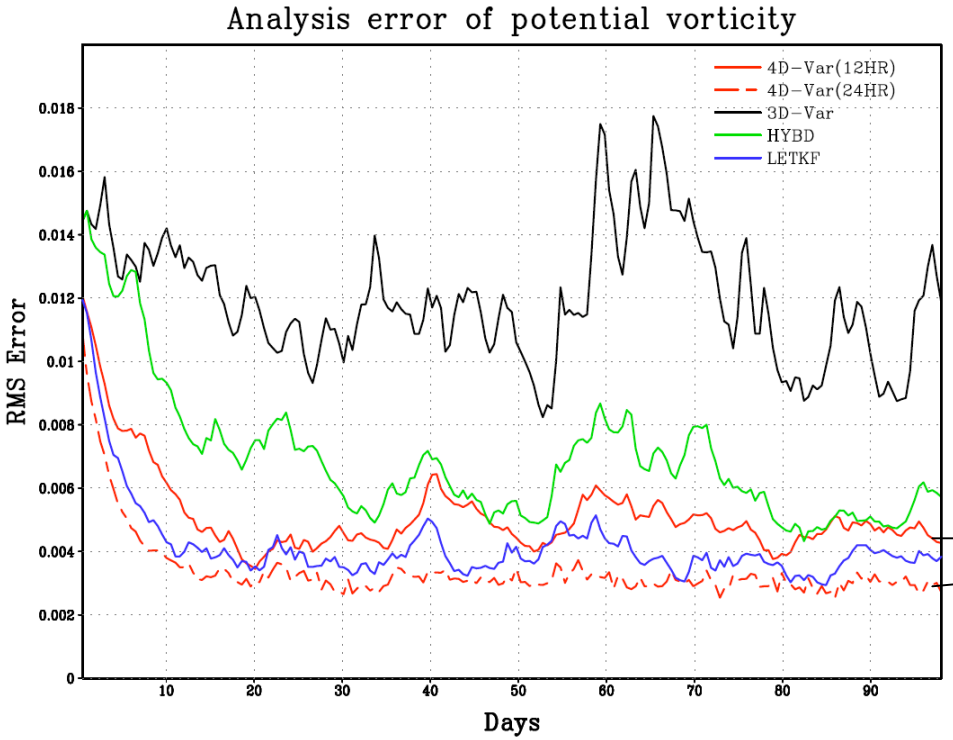
HYBD, 20 BV

HYBD, 20 BV, localized

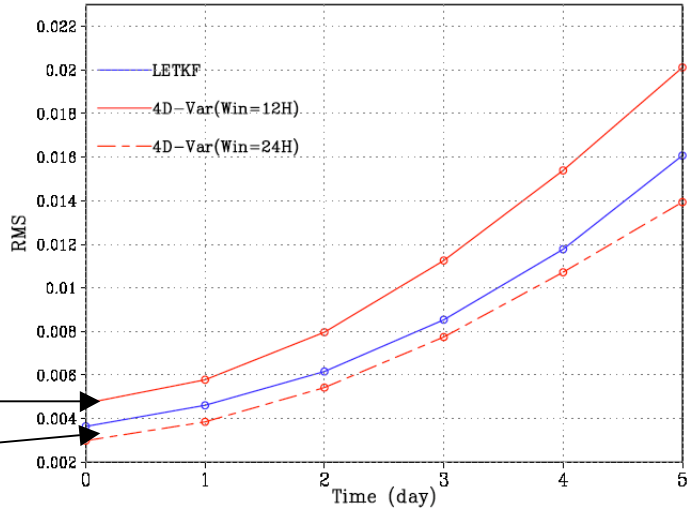
4D-Var

- The hybrid scheme performs better because of its ability to include the dynamically evolving errors
- By localizing the BVs, α increases and the hybrid scheme perform much better

RMS analysis/forecast errors



Forecast errors versus time



The performance of LETKF is better than 4D-Var with 12-hour but worse than 4D-Var with 24-hour window

Computational costs

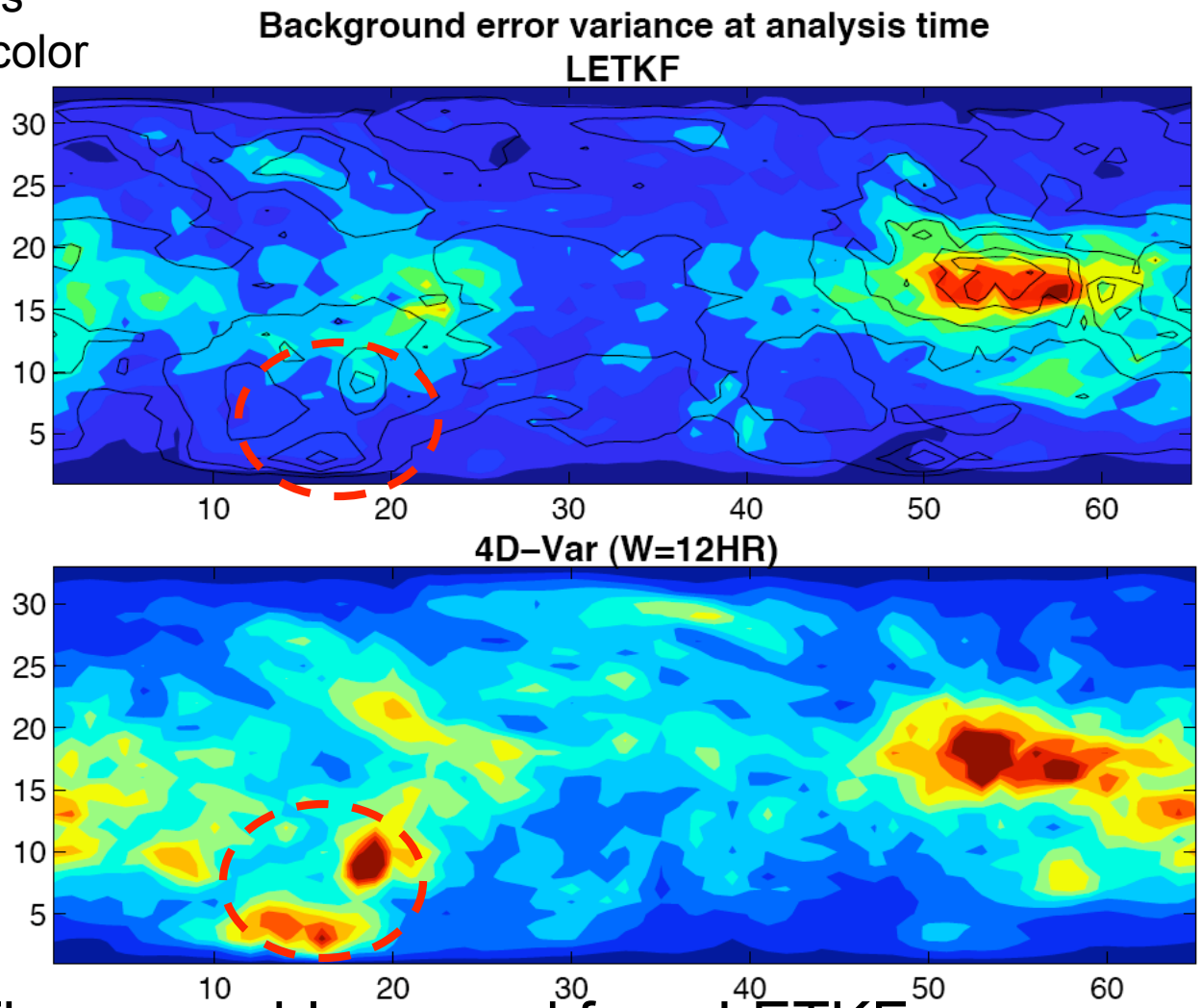
- Computational time with 1 CPU

	3D-Var	HYBD	4D-Var		LETKF		
			12HR	24HR	L=5	L=7	L=9
RMS error ($x10^{-2}$)	1.44	0.70	0.56	0.35	0.48	0.45	0.39
Time (hour)	0.5	5.0	**8.0**	**14.0**	**5.5**	**8.3**	**10.0**

LETKF can be computed in parallel

Error variance vs. ensemble spread

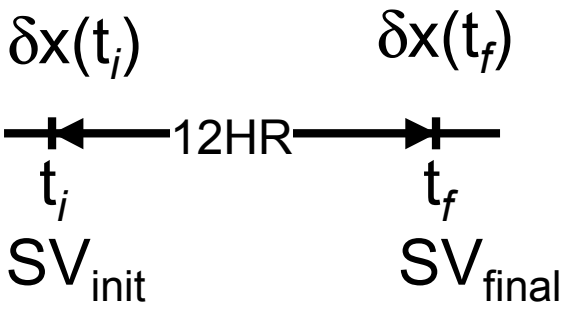
Spread: contours
Error variance: color



The ensemble spread from LETKF can represent the dynamically evolving error very well!

The structures of analysis increment

- 4D-Var analysis increments vs. singular vector(SV)
  - SV is defined with potential enstrophy norm with a chosen optimization time
  - Compared at initial/final time

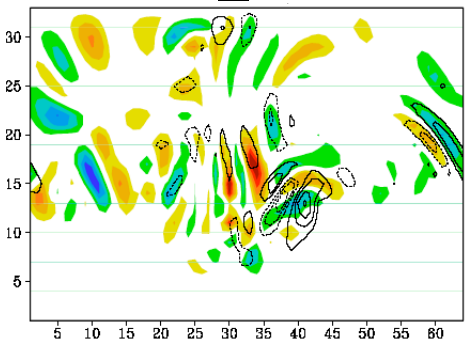


- LETKF analysis increment vs. bred vector(BV)
  - At the analysis time

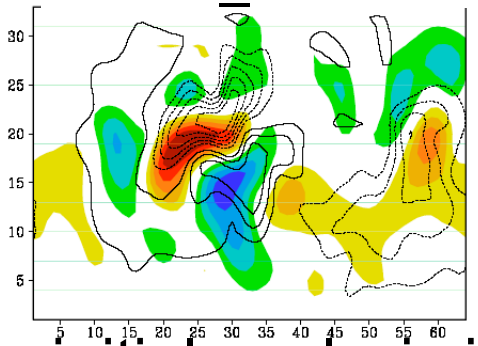
Structure of analysis increments

4D-Var 12-hour

final Ana_inc vs. SV1

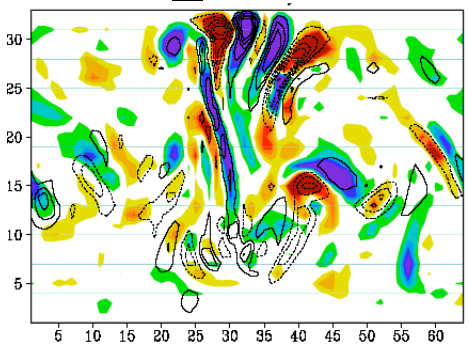


init Ana_inc vs. SV1

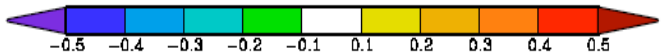


LETKF

Ana_inc vs. BV



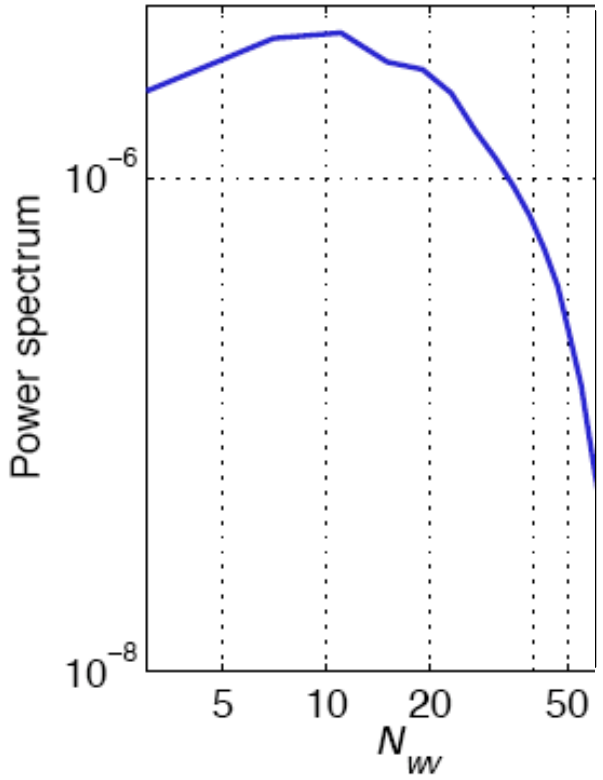
Ana_inc: color; **SV/BV**: contours



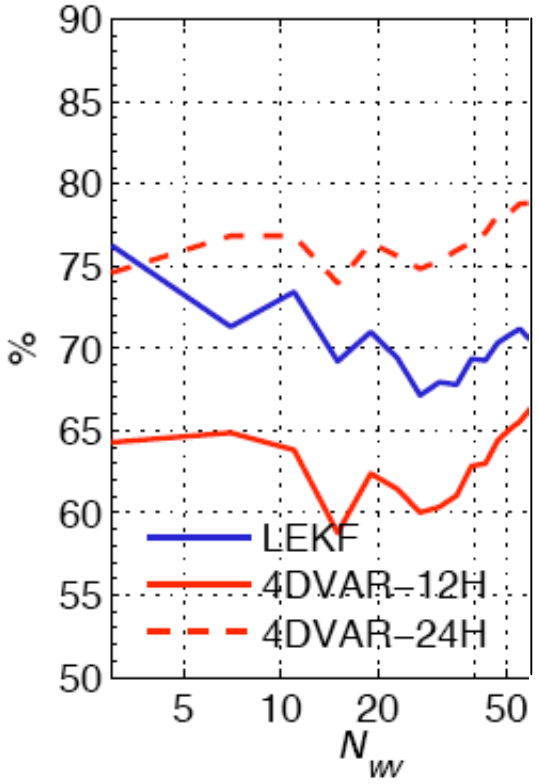
The initial analysis increments in 4D-Var are very different from the final increments, which are more similar to the analysis increments in LETKF

Relative improvement in spectral coordinates

3D-Var Analysis error of potential vorticity at z=3



Relative improvement with respect to 3D-Var



Summary

From the perfect model experiments with an analysis cycle of 12-hour, we show that

- The ensemble spread from LETKF is able to reflect well the error covariance structure.
- LETKF has the performance in between the results of 4D-Var with 12-hour and 24-hour window. 4D-Var has an advantage with a long window.
- The analysis increment from LETKF is very similar to the analysis increment of 4D-Var at the end of the assimilation window. Both strongly resemble the BV and final SV.
- Both LETKF and 4D-Var successfully improve the 3D-Var analysis in all scales. The improvement of LETKF of large scale is as good as the 4D-Var with 24-hour window.