RISE undergraduates find that regime changes in Lorenz's model are predictable

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Accepted in the Bulletin of the American Meteorological Society

July 2003

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The summer of 2002 marked the beginning of the Research Internships in Science and Engineering (RISE) program. RISE worked to build an extensive network of women faculty, science and engineering researchers, graduate students, and undergraduates. The program built this network through an eight-week summer research experience for "rising" junior and senior undergraduates. The goal was to encourage all participants to remain in science and engineering and to pursue graduate degrees.

By engaging twenty undergraduate junior and senior RISE scholars in teams with research projects coordinated by female faculty, the program introduced female students to women mentors and role models while providing high-quality opportunities to enhance their research knowledge and skills. RISE interns received advanced training in team skills, interpersonal communication, and project management. They were also able to become a part of the hierarchy of female mentorship by interacting with a group of incoming freshmen students. By sharing their experience as students in science and engineering and as RISE interns, they became role models to the younger students. One of the RISE participants described her experience as follows: "As an intern in the RISE program, my main expectation was to gain familiarity with the research process. Without prior research experience, I was unsure if graduate school was a realistic option for me. The RISE program allowed me to make my final decision to pursue a graduate degree and gave me confidence in my ability to contribute to a research project. I was assigned to a project in the field of Meteorology, a field about which I had little or no knowledge. Through the guidance of our faculty mentor, Dr. Eugenia Kalnay, and graduate advisors Malaquías Peña and Shu-Chih Yang, my team and I were able to first understand problems involved with weather prediction, and then apply our new knowledge to researching a method of weather prediction. Interpreting results, accepting that actual results may not agree with expected ones, and exploring new paths that the results lead to, are all exciting components of the creative process of research that my team and I had the opportunity to engage in. The fact that we were able to contribute to the discovery of new results helped me to decide to continue participating in research in graduate school." The summer concluded with the RISE Research Symposium, where research teams presented the results of their research. Coordinating faculty and representatives from the A. James Clark School of Engineering and the College of Mathematics and Physical Sciences, as well as staff from the National Science Foundation, were among those attending the Symposium. Dr. Rita Colwell, Director of the National Science Foundation, gave the keynote address.

One of the teams worked on atmospheric predictability. The purpose of this note is to describe the experience and results obtained by this team in order to encourage similar programs to attract women and minorities into graduate studies in the geosciences. Although the four RISE interns were selected because of their outstanding mathematical, physical and computer sciences skills, three of them had no background in meteorology, and the fact that the research internship had to be completed in 8 weeks imposed a significant challenge. The team was given a problem: become familiar with the famous Lorenz (1963) model, and explore its predictability using breeding (Toth and Kalnay 1997, Kalnay, 2003), an algorithm chosen for this project because of its simplicity. The Lorenz model equations are

$$\begin{aligned}\frac{dx}{dt} &= \sigma(y-x) \\ \frac{dy}{dt} &= rx - y - xz \\ \frac{dz}{dt} &= xy - bz\end{aligned} \qquad (1)$$

where the parameters $\sigma = 10$, $b = 8/3$, and $r = 28$ chosen by Lorenz result in chaotic solutions (Fig. 1). This model has been very widely used as a prototype of chaotic behavior and an example of lack of long-term predictability (e.g., Sparrow, 1982, Tsonis, 1992, Kalnay et al., 2002). The stability properties and the dependence of the forecast error growth on the initial conditions have been previously studied (e.g., Nicolis et al, 1983, Nese, 1989, Elsner and Tsonis, 1992, Palmer, 1993), but we are not aware of studies about the prediction of the occurrence of regime changes and their duration. The students were given as a template a MATLAB program of a coupled fast-slow Lorenz model written by Jim Hansen, from which they unraveled the classic Lorenz model code and learned how to run and plot its results. They were asked: "Imagine that you are a forecaster living in the Lorenz attractor. Everybody in the attractor knows that there are two weather regimes, which we could denote as 'Warm' and 'Cold' (see Figure 1), but the public needs to know *when* changes in regime will happen and *how long* will they last. Can you develop simple forecasting rules to alert people about imminent changes of regime?"

The students implemented breeding, a method used to estimate forecast errors in weather models. Bred vectors are simply the difference between two model runs, the second originating from slightly perturbed initial conditions, periodically rescaled (Figure 2). The amplification of the bred vectors can be used to identify regions of high error growth within the attractor. The Lorenz model used in this project was integrated using a fourth order Runge-Kutta time scheme with a time step of Δt=0.01. The bred vectors were obtained from a second run with the same model started from an initial perturbation $\delta\mathbf{x}_0 = (\delta x_0, \delta y_0, \delta z_0)$ added to the control at time t_0. Every 8 time steps the vector difference $\delta\mathbf{x}$ between the perturbed and the control run was rescaled to the initial amplitude $|\delta\mathbf{x}_0| = \sqrt{\delta x_0^2 + \delta y_0^2 + \delta z_0^2}$ and added to the control run (Fig. 2). The bred vector amplification factor was defined as the size of the bred vector after $n = 8$ steps divided by its original size $|\delta\mathbf{x}|/|\delta\mathbf{x}_0|$, and the growth rate as $g = \frac{1}{n}\ln\left(|\delta\mathbf{x}|/|\delta\mathbf{x}_0|\right)$. The students plotted the observed bred vector growth on the Lorenz attractor in order to explore its predictability (Figure 3). Red indicates that during the last 8 steps the perturbation growth rate *g* was larger than 0.064 (i.e., the size of the bred vectors grew by 1.67 or more in 8 time steps), whereas blue indicates a negative growth rate, meaning that the perturbations are actually decaying. The results shown in this figure were very promising because they suggested that bred vector growth would allow estimating regions of high and low predictability of the attractor.

The students then examined the bred vector growth for patterns of predictability. They found that plotting the growth rates on the evolution of the variable *x(t)* provides a means to predict when the model will enter a new regime, and also how long the new regime will last. Figure 4 illustrates the "forecasting rules" that the students developed by inspection. **Rule 1:** When the growth rate exceeds 0.064 over a period of 8 steps, as indicated by the presence of one (or more) red stars, the current regime will end after it completes the current orbit. **Rule 2:** The length of the new regime is proportional to the number of red stars. For example, the presence of 5 or more stars in the old regime, indicating sustained strong growth, implies that the new regime will last 4 orbits or more (see Fig. 5 for the relationship between number of red stars and the duration of the new regime).

After the RISE internship had been completed, and the results presented at the RISE Research Symposium, Evans (supported by the School of Engineering) and Peña carried out an objective verification of these simple forecasting rules. Table 1 is the contingency table for the categorical Rule 1 that forecasts the occurrence of a regime change during the following orbit. Table 2 is the corresponding contingency table for Rule 2 that the presence four stars or less indicates that the new regime will only last up to three orbits. Figure 5 shows that there is a strong relationship between the number of red stars in the old regime and the duration of the new regime.

The verification scores obtained for the rules, with hit rates over 90%, threat scores over 80%, and false alarm rates of less than 10% indicate that both rules provide excellent predictions of regime change and duration.

In summary, the RISE students succeeded in providing the mythical inhabitants of the chaotic Lorenz attractor with robust prediction rules that would allow them to be prepared for changes in regime and indicate how long the new regime would probably last. While in this process, the undergraduate women learned that they were able to both perform and enjoy research and strengthened their motivation to pursue research careers in science and engineering.

Acknowledgements: The RISE program was funded by the National Science Foundation, the A. James Clark School of Engineering, and the University of Maryland, and coordinated by the Women in Engineering Program. We are very grateful to Linda Schmidt, Janet Schmidt and Paige Smith for conceiving and obtaining funds for the RISE program, for the support of the National Science Foundation and the University of Maryland, and for the extension of support to Erin Evans.

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Figure legends:

Figure 1: Solution of the Lorenz model equations (1) over 1500 time steps, showing a “warm” regime with positive values of x and y, and a “cold” regime with negative values of x and y. The solution typically remains for several loops in each regime before changing to the other regime.

Figure 2: Schematic of the construction of bred vectors, which are the difference between a perturbed and a “control” (unperturbed) solution. Every few (in this case 8) steps, the difference, rescaled to the original size and added to the control forecast, becomes the initial condition for the perturbed forecast. The ratio between the initial and the final size is the amplification of the bred vector during that interval.

Figure 3: The Lorenz “butterfly” attractor with bred vector growth over 8 steps. The table indicates the range of the growth rate corresponding to each color.

Figure 4: Time series of the variable x versus time step, with breeding cycles of 8 time steps, with colored stars indicating the bred vector growth as in Fig. 3. Each panel shows 4000 steps, corresponding to 500 breeding cycles.

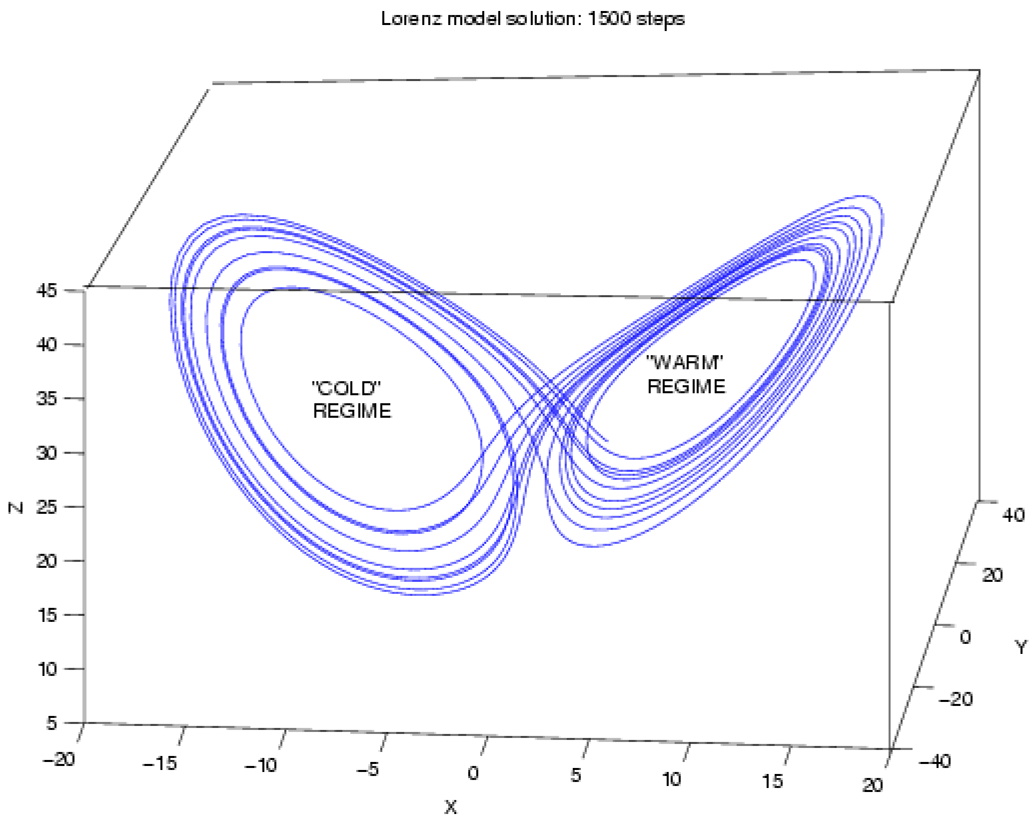
Figure 5: Observed number of cycles in the new regime for a given number of red stars in the old regime. The numbers indicate the number of pairs observed, with blanks indicating no observed pairs. Dashed lines indicate the specific rule verified in Table 2, although other combinations yield similarly high verification scores.

Obs / Fcst	Yes	No	Total
Yes	187	13	200
No	33	299	332
Total	220	312	532

Table 1: Contingency table for Rule 1 (a change of regime takes place in the orbit after the appearance of at least one red star), computed over 40,000 time steps, with 187 changes of regime. These numbers correspond to a hit rate (percentage of the forecasts correctly anticipating the subsequent change or lack of change of regime) is HR=91.4%, the threat score or critical success index is TS=80.3%, and the false alarm rate, the percentage of forecasts in which a change of regime was forecast but did not occur, is FAR=6.5% (Wilks, 1995, pp 238-241).

Obs / Fcst	Yes	No	Total
Yes	134	3	137
No	12	38	50
total	146	41	187

Table 2: Contingency table for Rule 2 (fewer than five red stars in the old regime indicate that the new regime will only last three orbits or less, see Figure 3), computed over 40,000 steps. These numbers correspond to HR=92.0%, TS=90.0%, and FAR=2.2%.



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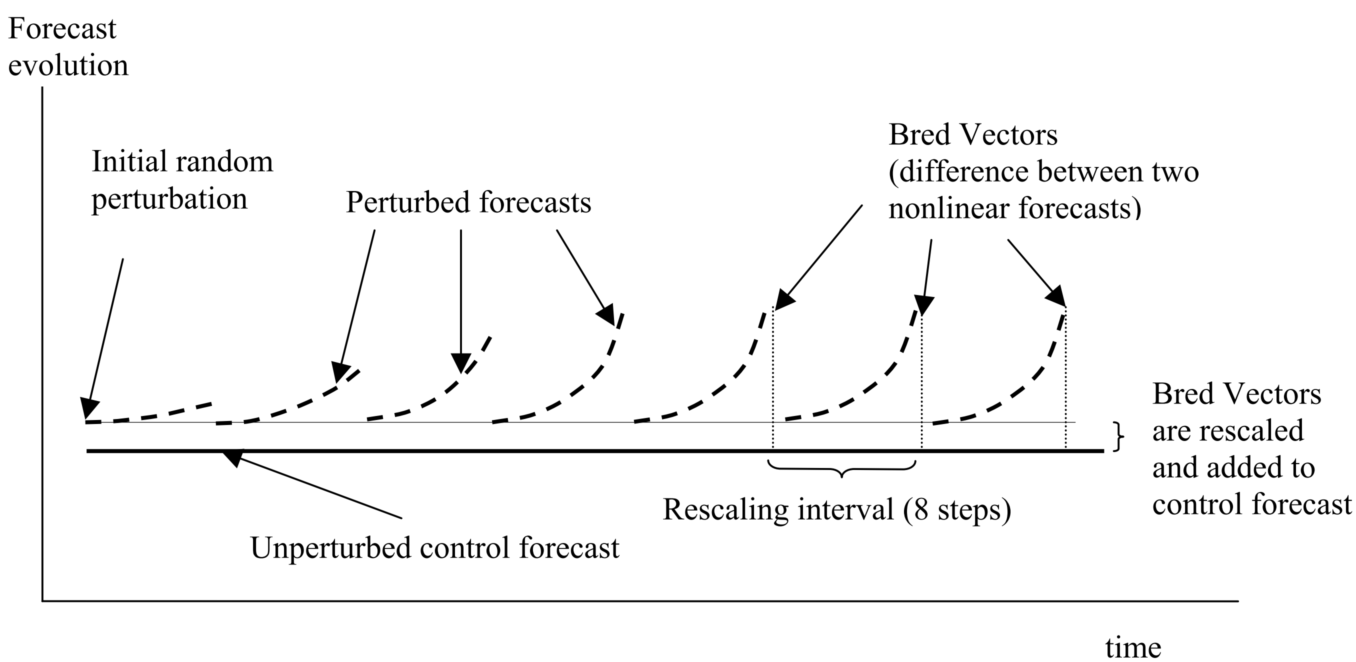

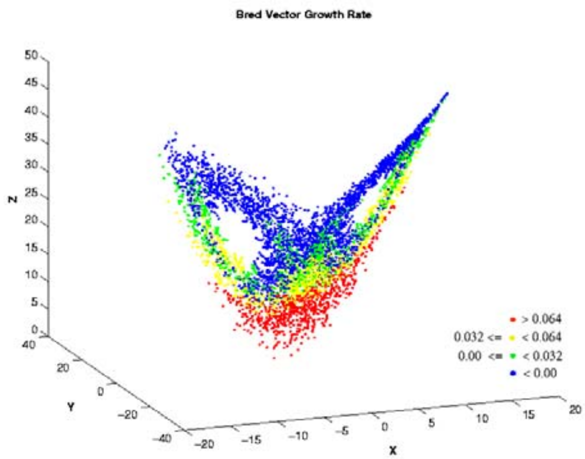

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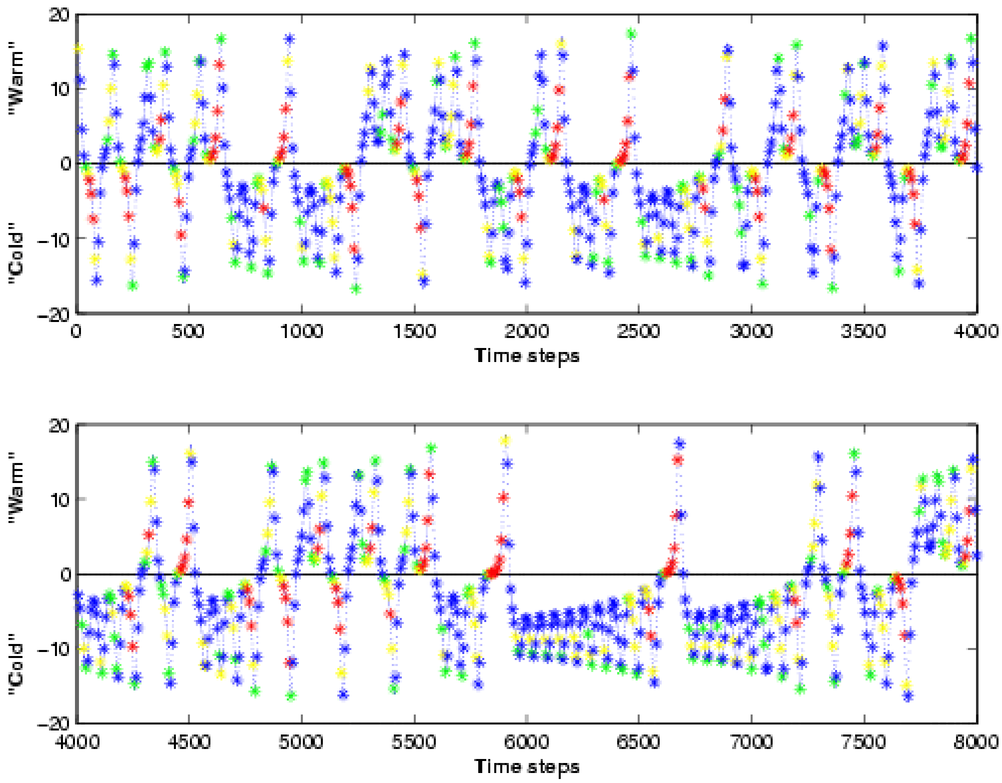

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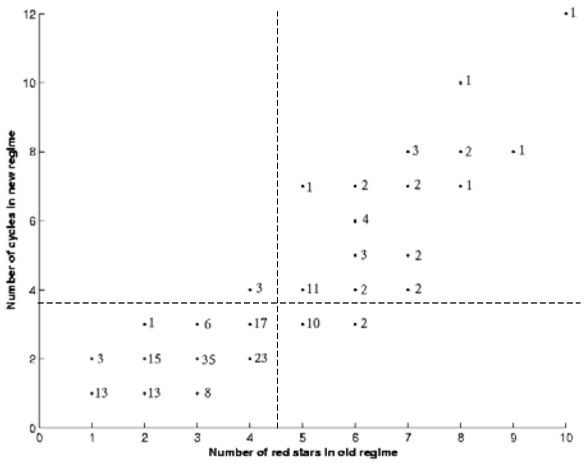

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