

# Chapter 6: Ensemble Forecasting and Atmospheric Predictability

## Introduction

# Deterministic Chaos (what!?)

In 1951 Charney indicated that forecast skill would break down, but he attributed it to model errors and errors in the initial conditions...

In the 1960's the forecasts were skillful for only one day or so.

Statistical prediction was equal or better than dynamical predictions,

Like it was until now for ENSO predictions!

Lorenz wanted to show that statistical prediction could not match prediction with a nonlinear model for the Tokyo (1960) NWP conference

So, he tried to find a model that was not periodic (otherwise stats would win!)

He programmed in machine language on a 4K memory, 60 ops/sec Royal McBee computer

He developed a low-order model (12 d.o.f) and changed the parameters and eventually found a nonperiodic solution

Printed results with 3 significant digits (plenty!)

Tried to reproduce results, went for a coffee and OOPS!

Lorenz (1963) discovered that even with a **perfect model** and **almost perfect initial conditions** the forecast loses all skill in a **finite time interval**: “A butterfly in Brazil can change the forecast in Texas after one or two weeks”.

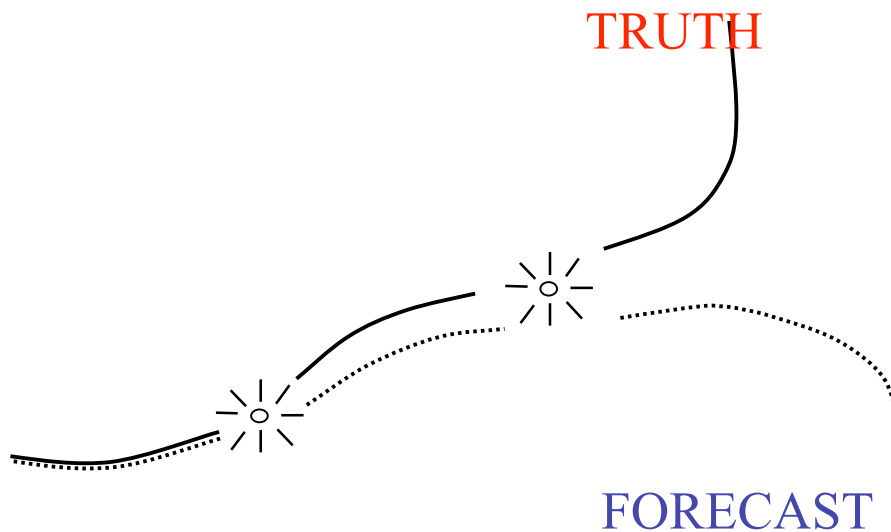
In the 1960's this was only of academic interest: forecasts were useless in two days

Now, we are getting closer to the **2 week limit of predictability**, and we have to extract the maximum information

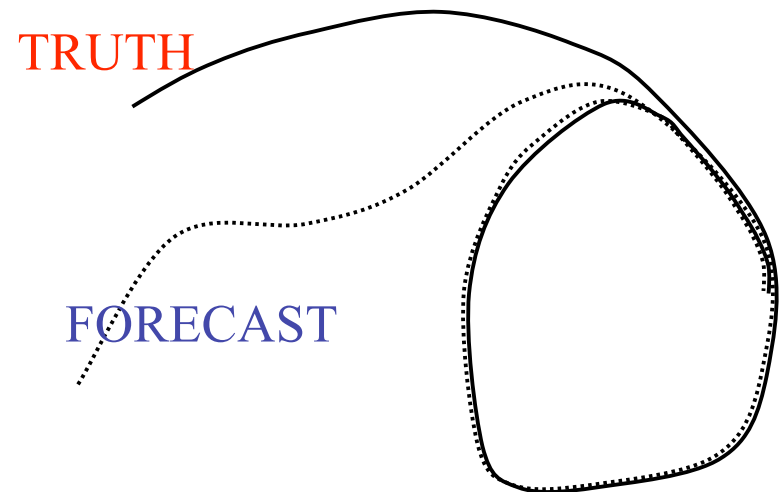
## Central theorem of chaos (Lorenz, 1960s):

- a) **Unstable** systems have **finite predictability** (chaos)
- b) **Stable** systems are **infinitely predictable**

a) Unstable dynamical system



b) Stable dynamical system



**A simple chaotic model:  
Lorenz (1963) 3-variable model**

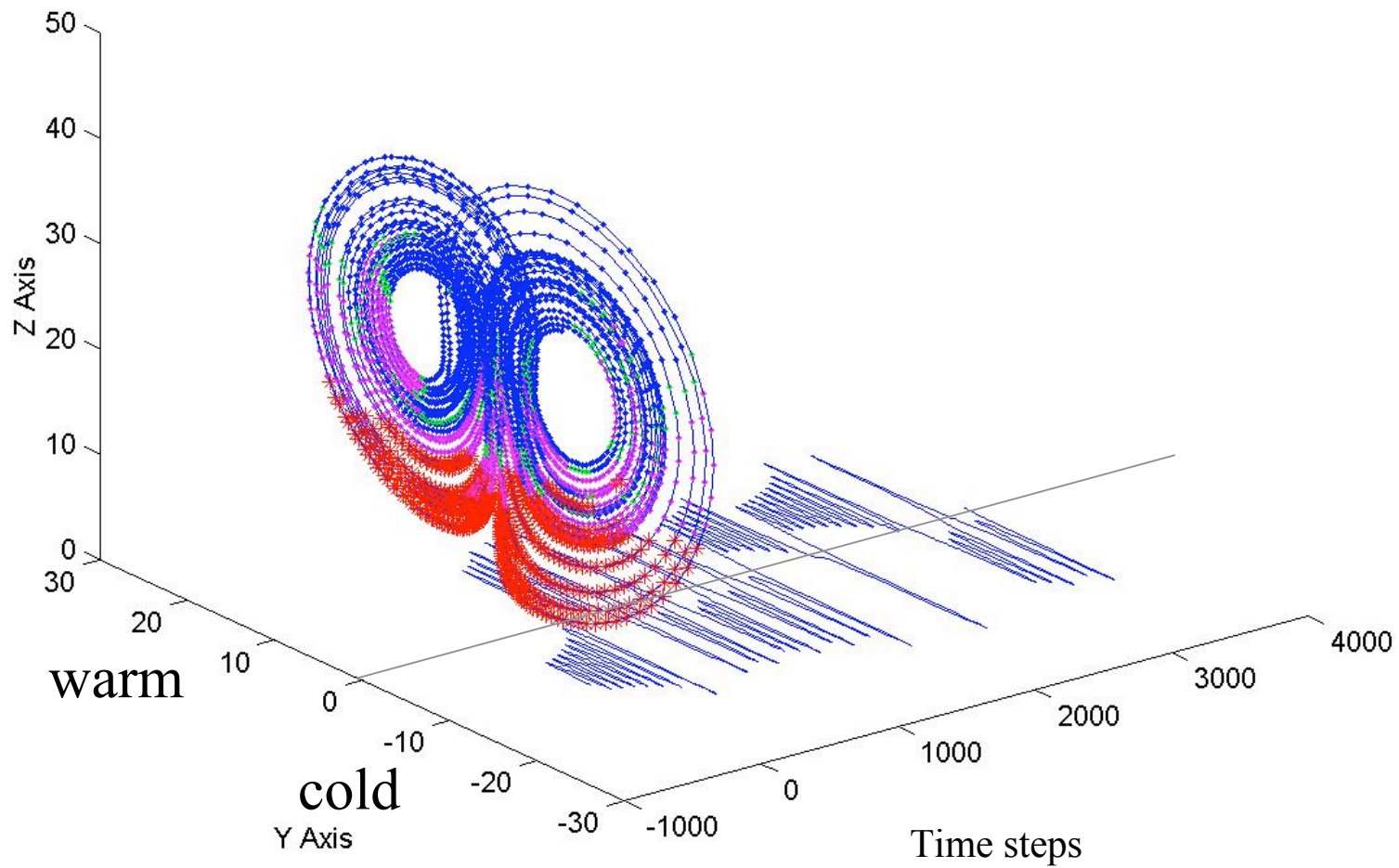
Has two regimes and the transition between them is  
chaotic

$$\frac{dx}{dt} = \sigma(y - x)$$

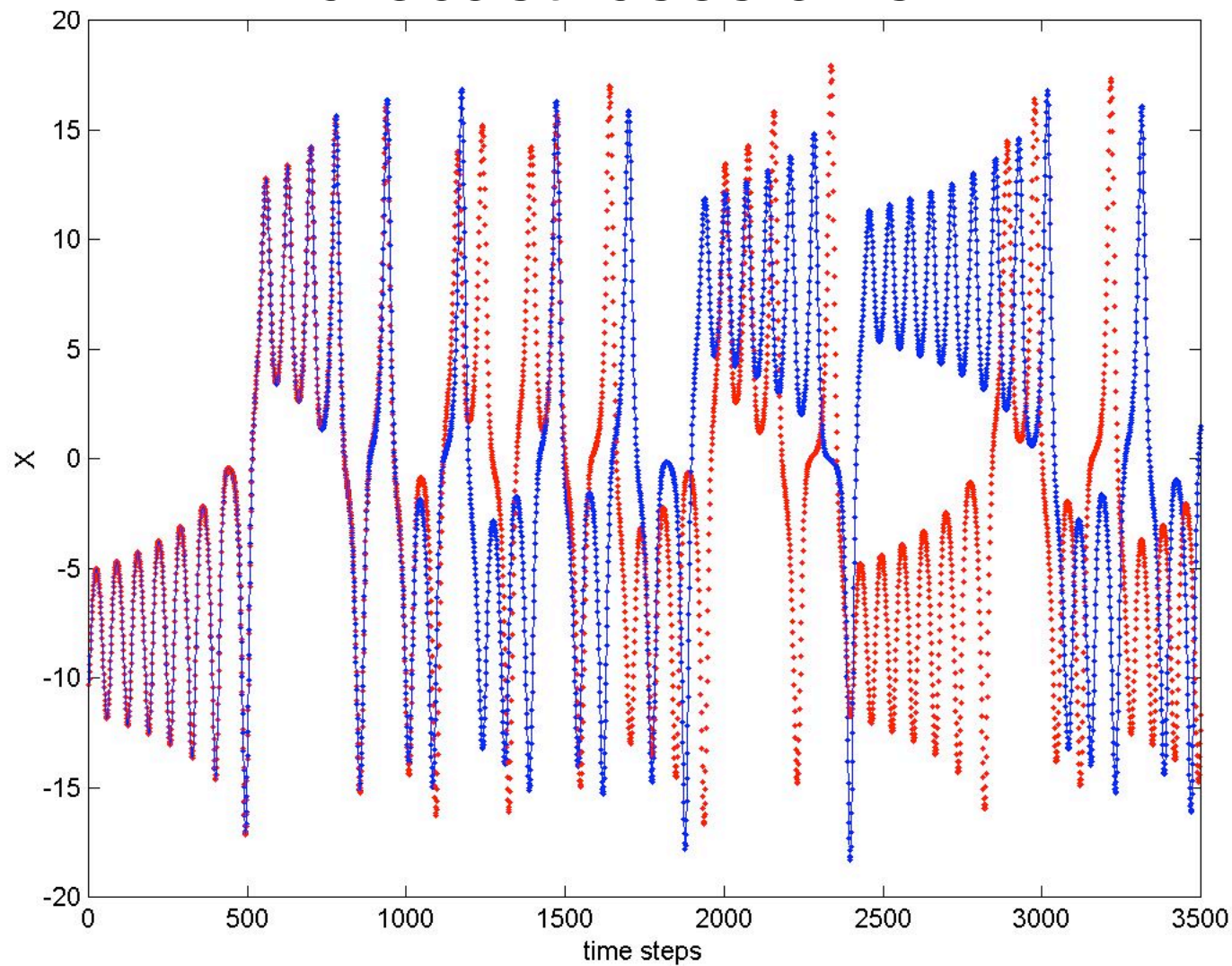
$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$

# Example: Lorenz (1963) model, $y(t)$



We introduce an infinitesimal perturbation in the initial conditions and soon the forecast loses all skill





# Definition of Deterministic Chaos

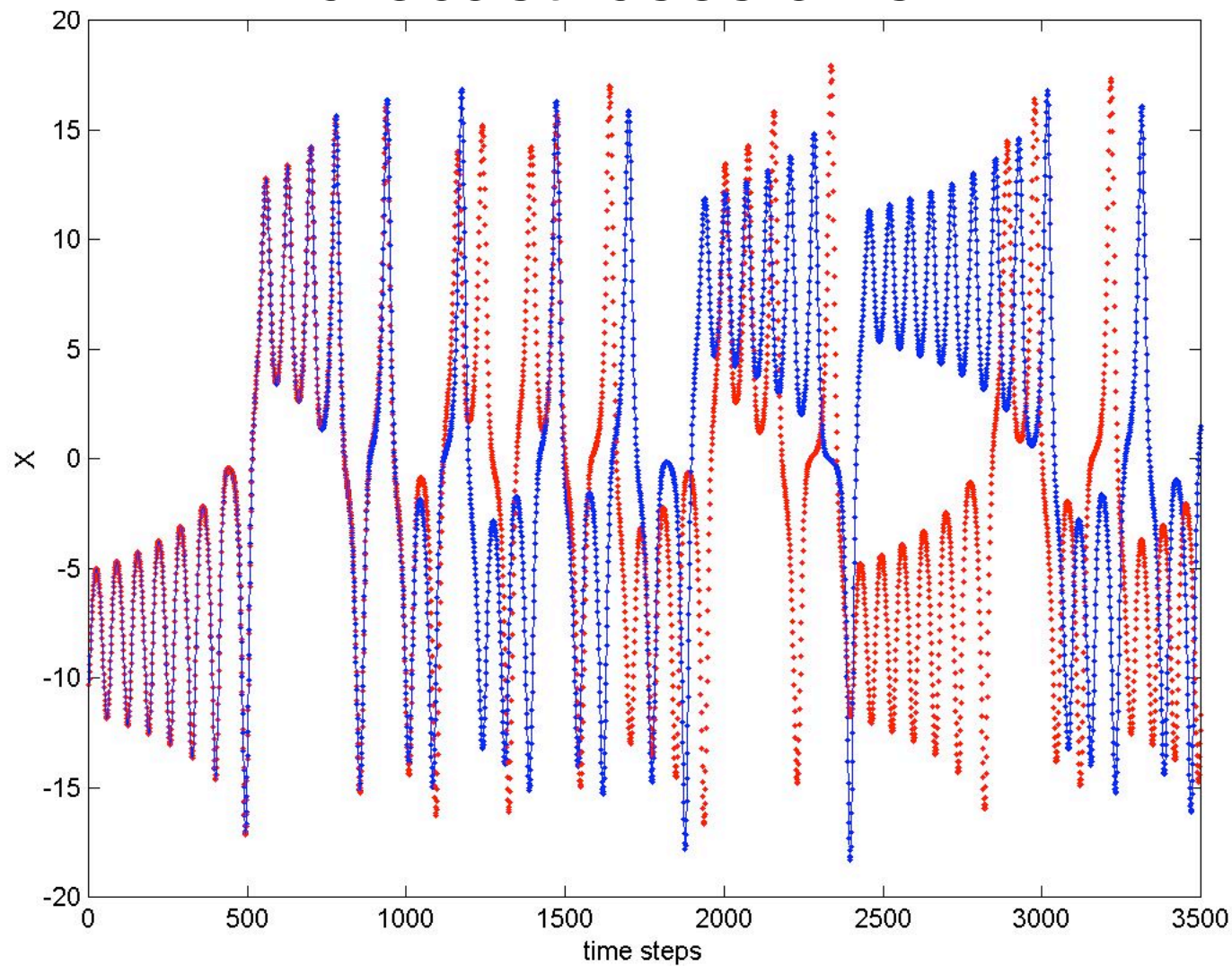
(Lorenz, March 2006, 89)

**WHEN THE PRESENT DETERMINES  
THE FUTURE**

**BUT**

**THE APPROXIMATE PRESENT DOES NOT  
APPROXIMATELY DETERMINE THE FUTURE**

We introduce an infinitesimal perturbation in the initial conditions and soon the forecast loses all skill



Predictability depends on the initial conditions (Palmer, 2002):

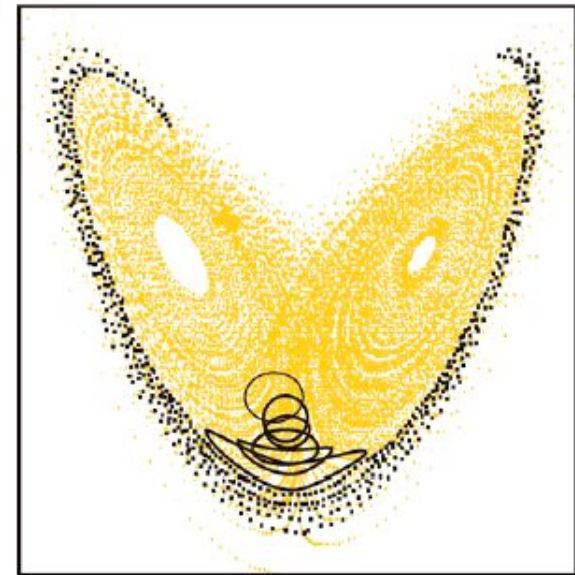
stable



less stable



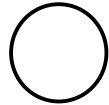
unstable



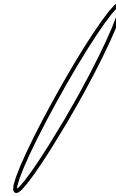
A “ball” of perturbed initial conditions is followed with time. Errors in the initial conditions that are unstable (with “errors of the day”) grow much faster than if they are stable

Fig. 6.2: Schematic of the evolution of a small spherical volume in phase space in a bounded dissipative system.

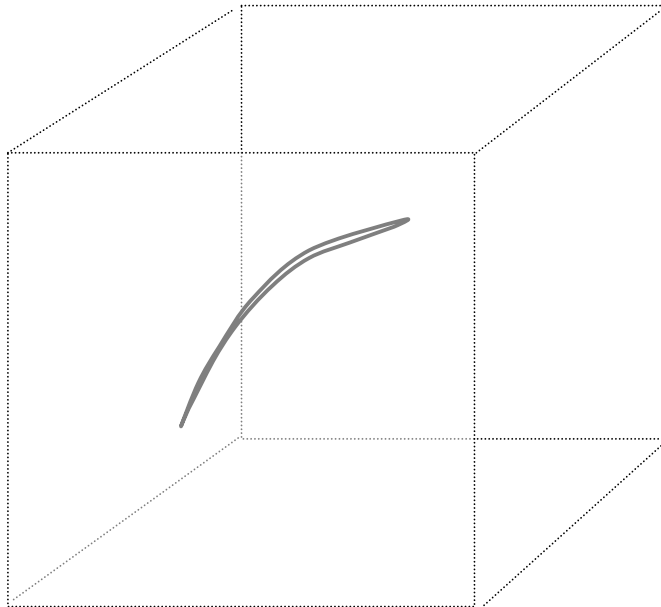
a) Initial volume: a small hypersphere



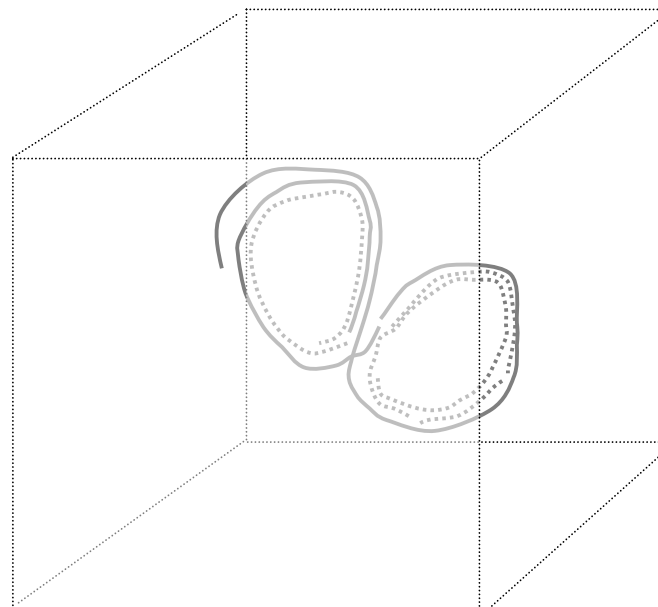
b) Linear phase: a hyper ellipsoid



c) Nonlinear phase: folding needs to take place in order for the solution to stay within the bounds



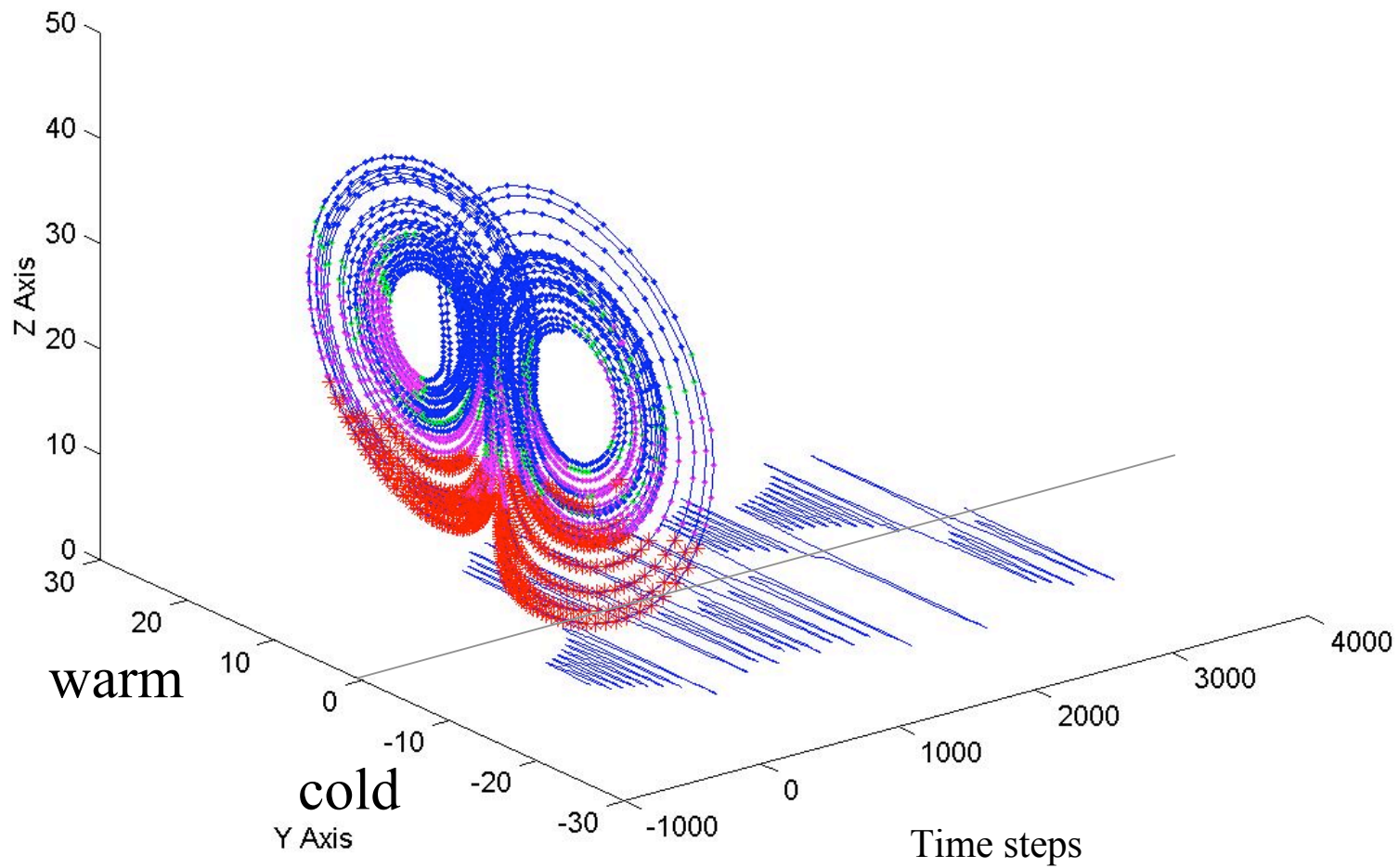
d) Asymptotic evolution to a strange attractor of zero volume and fractal structure. All predictability is lost



## An 8 week RISE project for undergraduate women

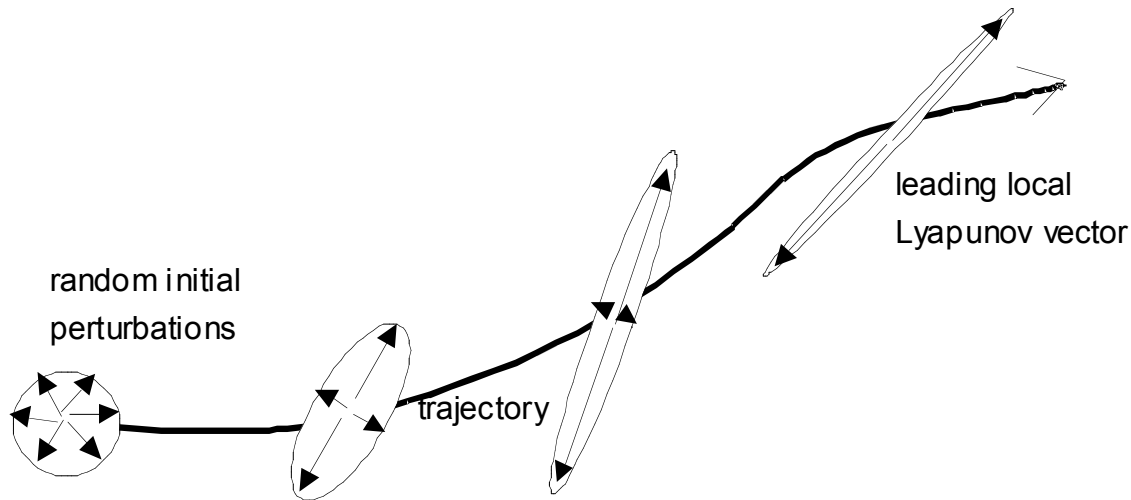
- We gave a team of 4 RISE intern undergraduates a problem: Play with the famous Lorenz (1963) model, and explore its predictability using “breeding” (Toth and Kalnay 1997), a very simple method to grow errors.
- We told them: “Imagine that you are forecasters that live in the Lorenz “attractor”. Everybody living in the attractor knows that there are two weather regimes, the ‘Warm’ and ‘Cold’ regimes. But what the public needs to know is **when** will the change of regimes take place, and **how long** are they going to last!!”.
- “Can you find a forecasting rule to alert the public that there is an imminent change of regime?”

# Example: Lorenz (1963) model, $y(t)$



When there is an instability, all perturbations converge towards the fastest growing perturbation (leading Lyapunov Vector). The LLV is computed applying the linear tangent model on each perturbation of the nonlinear trajectory

Fig. 6.7: Schematic of how all perturbations will converge towards the leading Local Lyapunov Vector

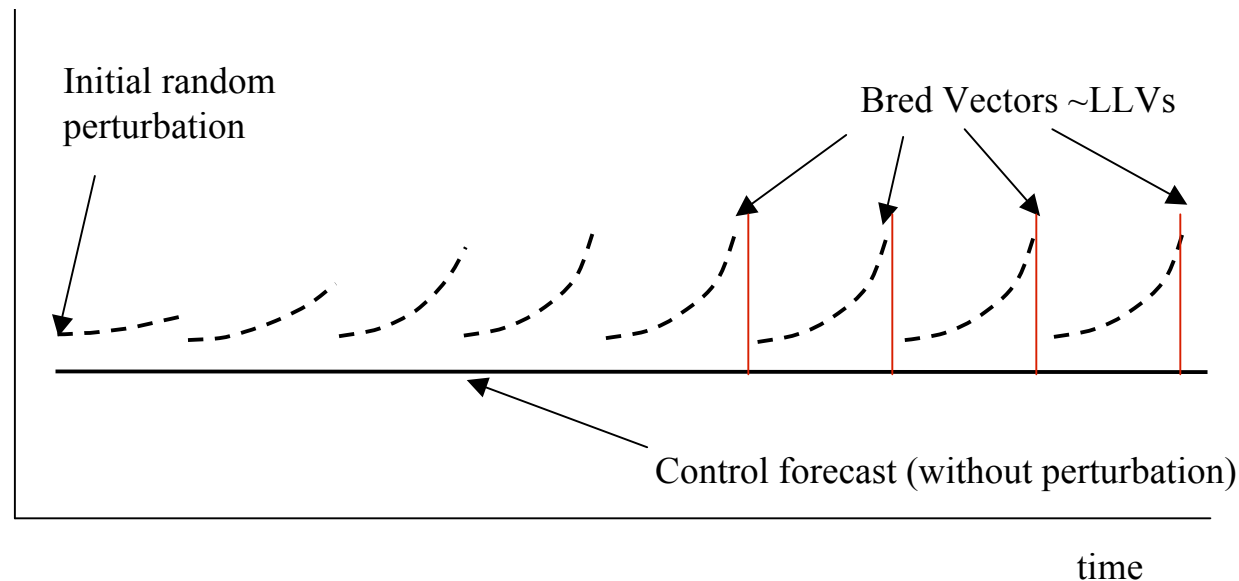


# “Breeding”: Grow naturally unstable perturbations, similar to Lyapunov vectors but using the nonlinear model twice

- Breeding is simply running the nonlinear model a second time, starting from perturbed initial conditions, rescaling the perturbation periodically

$$g(t) = \frac{1}{n\Delta t} \ln \left( \frac{|\delta \mathbf{x}|}{|\delta \mathbf{x}_0|} \right)$$

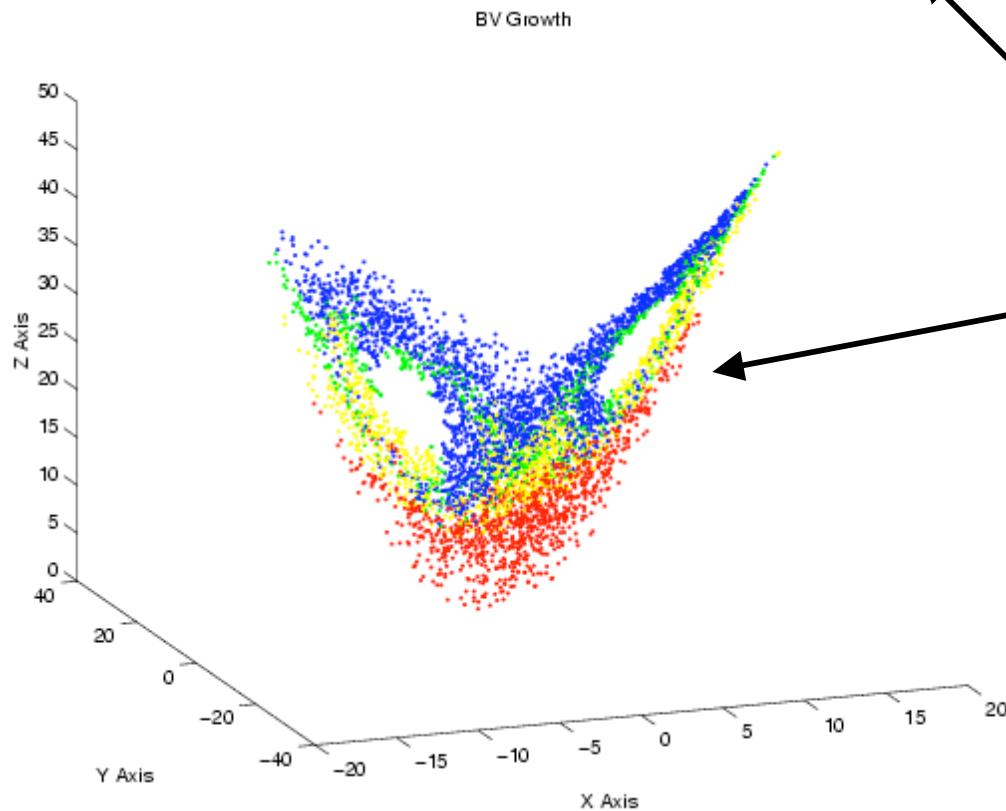
Forecast values





In the 3-variable Lorenz (1963) model we used 'breeding'  
the local growth of the perturbations:

Growth of the bred vectors: red: large; yellow,  
medium; green, low; blue: negative (decay)



We "painted"  
the trajectory  
with the  
growth rate

With a simple breeding cycle we were able to estimate  
the stability of the attractor (Evans et al, 2003)

Predictability depends on the initial conditions (Palmer, 2002):

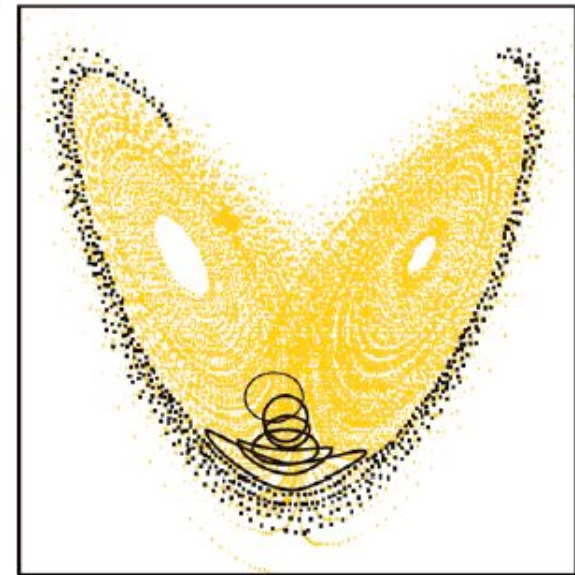
stable



less stable

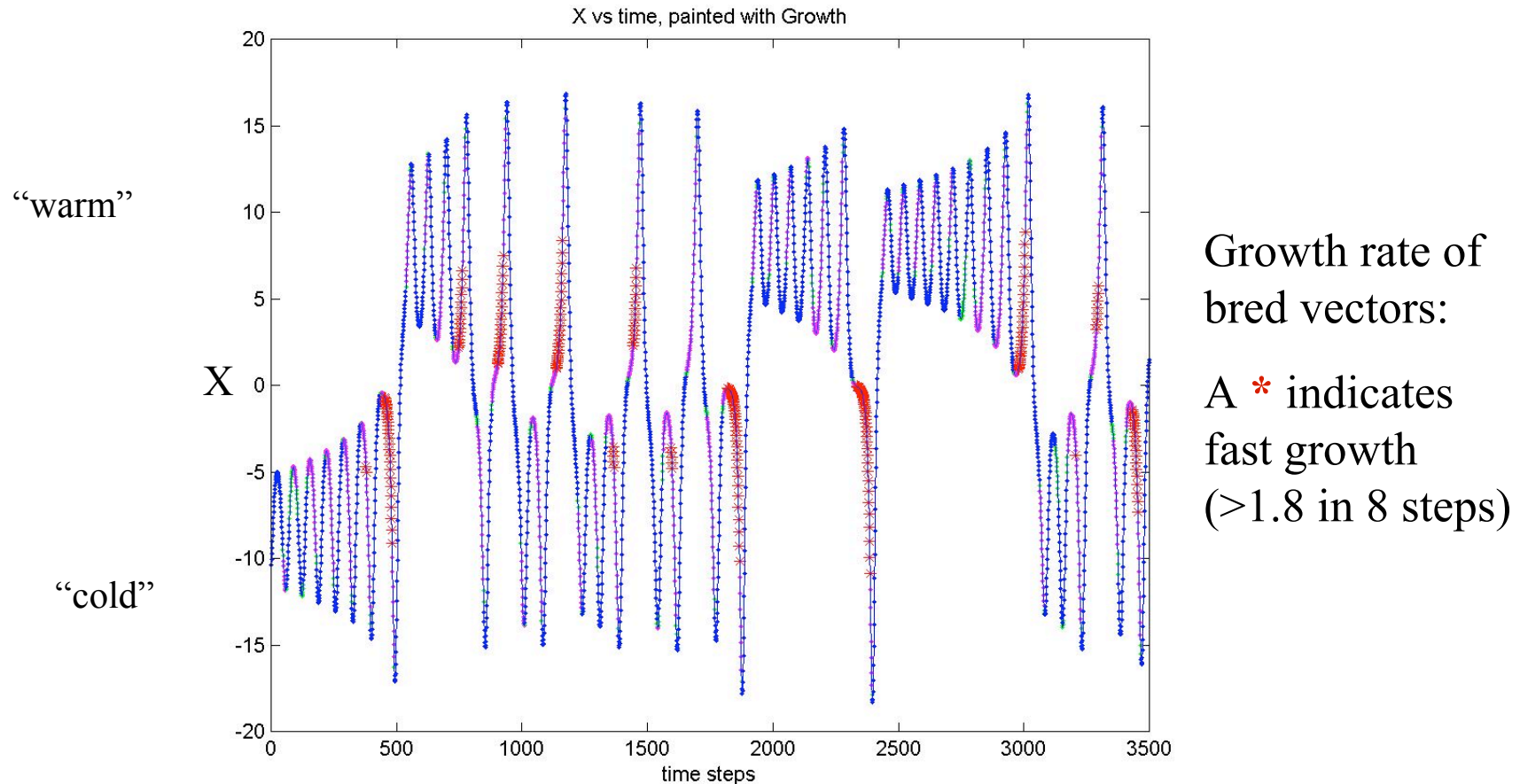


unstable



Initial conditions that are unstable (with “errors of the day”) grow much faster

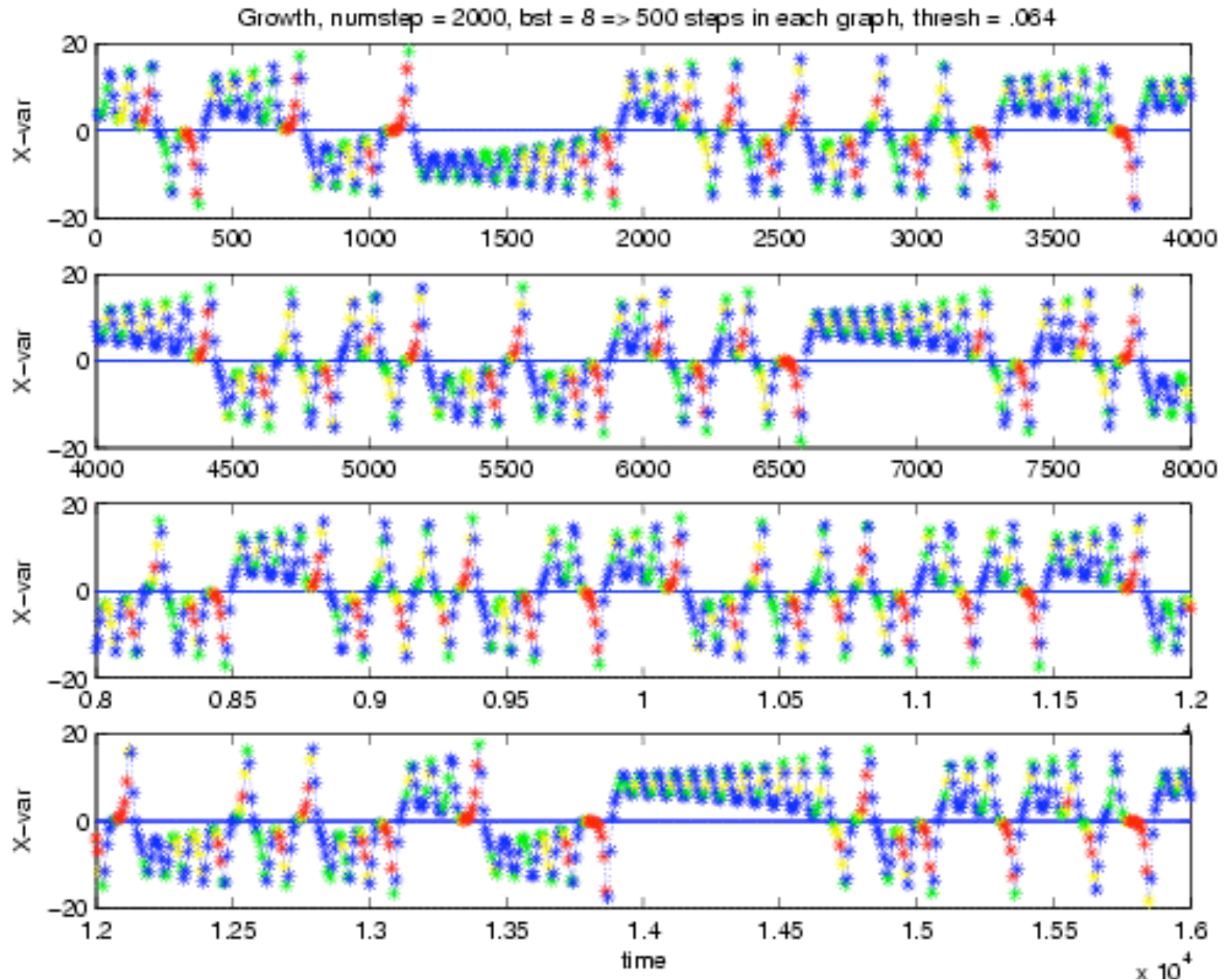
# Rules for a forecaster living in the Lorenz model:



1) Change of Regime: The presence of **red stars** indicates that the next orbit will be the **last orbit in the present regime**.

2) Duration of the New Regime: **Few red stars**, the next regime will be short. **Many red stars**: the next regime will be long lasting.

These are very robust rules, with skill scores  $> 95\%$



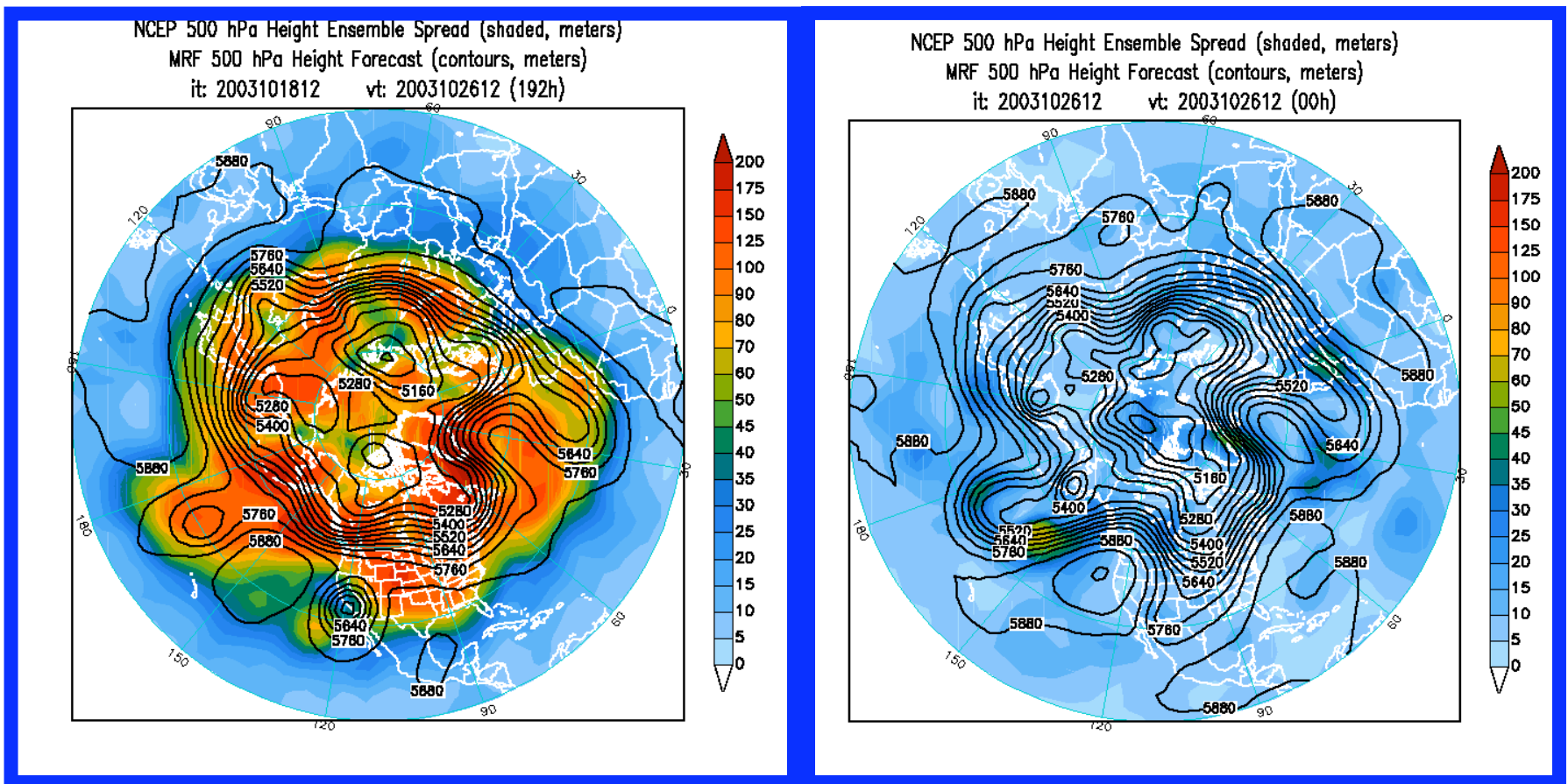
# Summary for this part

- Breeding is a simple generalization of Lyapunov vectors, for finite time, finite amplitude: simply run the model twice...
- The only parameters are the amplitude and the frequency of the renormalization (it does not depend on the norm)
- Breeding in the Lorenz (1963) model gives forecasting rules for change of regime and duration of the next regime that surprised Lorenz himself...

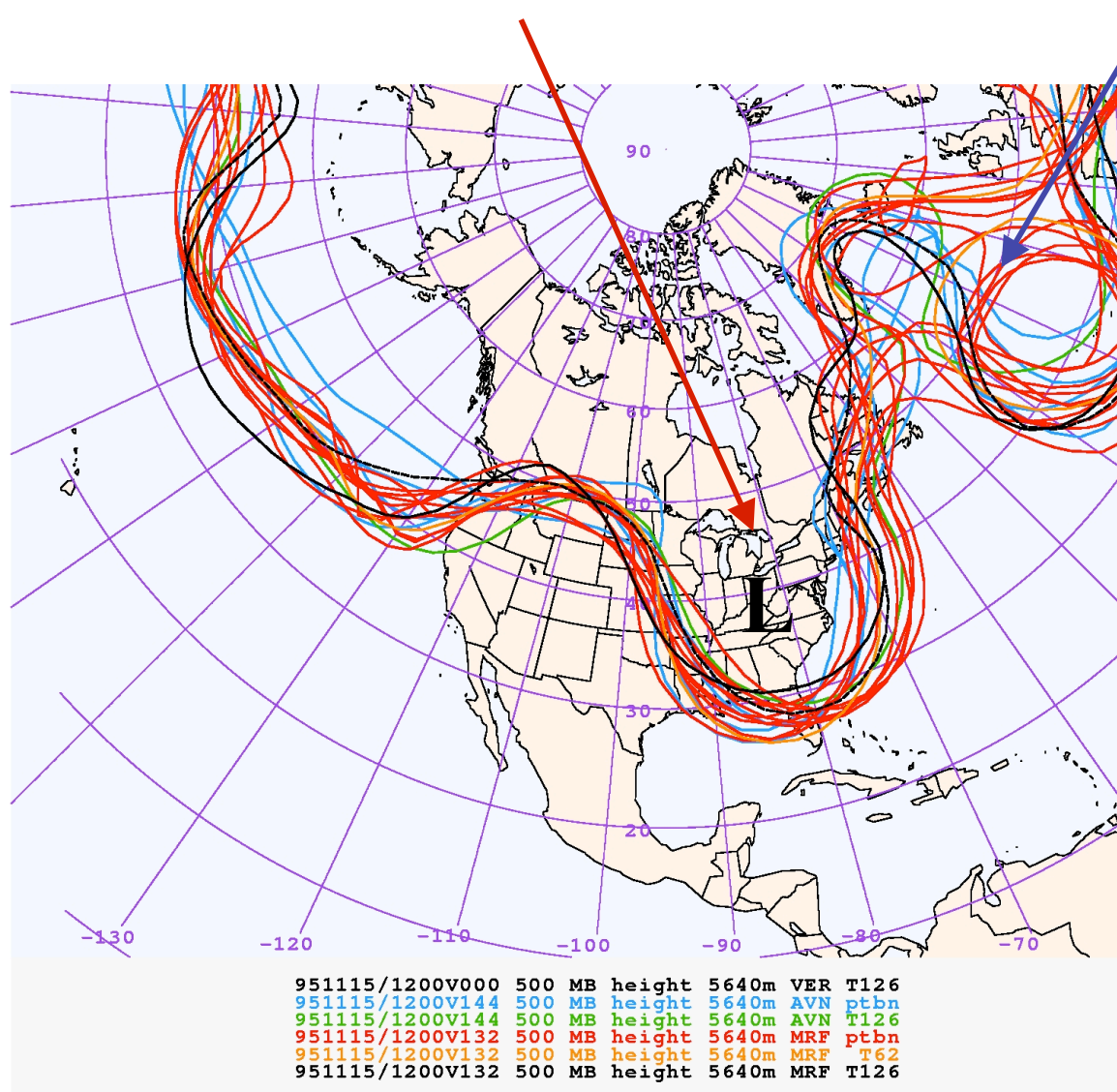
# Ensemble Forecasting

- It used to be that a single control forecast was integrated from the analysis (initial conditions)
- In ensemble forecasting several forecasts are run from slightly perturbed initial conditions (or with different models)
- The spread among ensemble members gives information about the forecast errors

8-day forecast and verification: for a “spaghetti” plot, we draw only one contour for each ensemble member forecast, showing where the centers of high and low pressure are



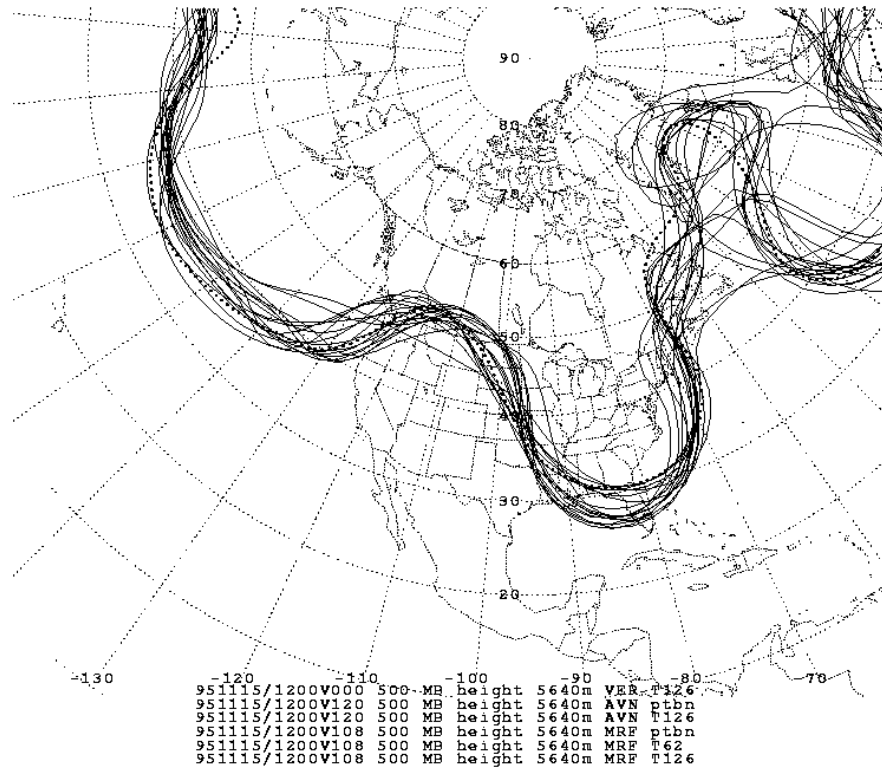
Example of a **very predictable 6-day forecast**, with “errors of the day”



Errors of the day tend to be localized and have simple shapes  
(Patil et al, 2001)

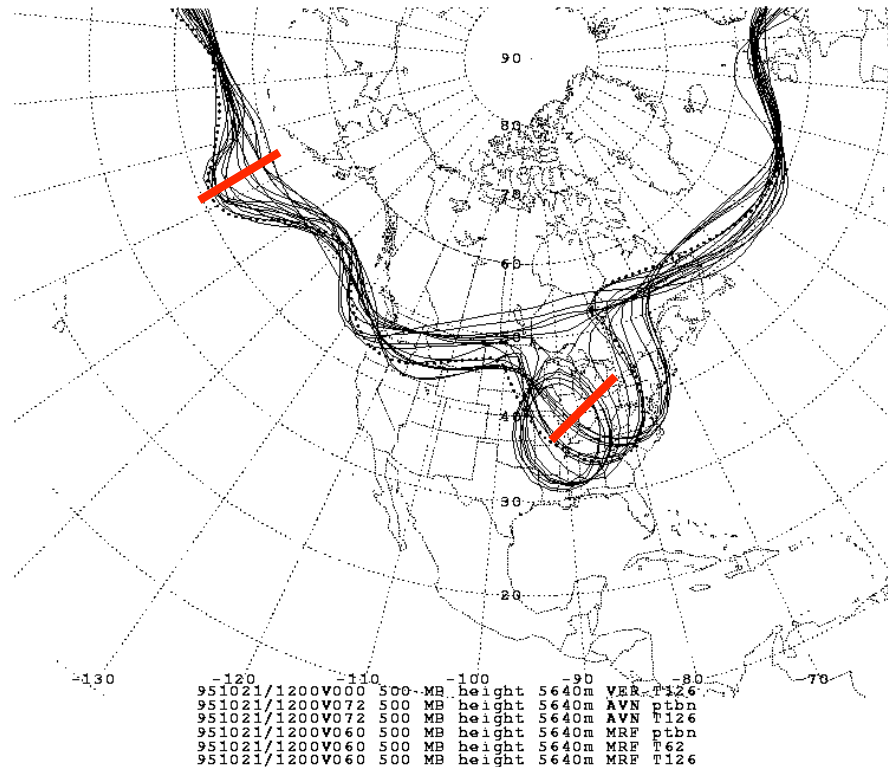


The errors of the day are instabilities of the background flow. At the same verification time, the forecast uncertainties have *the same shape*



4-day forecast  
verifying on  
the same day

Strong instabilities of the background tend to have simple shapes (perturbations lie in a low-dimensional subspace)



2.5 day forecast verifying on 95/10/21.

Note that the bred vectors (difference between the forecasts) lie on a 1-D space

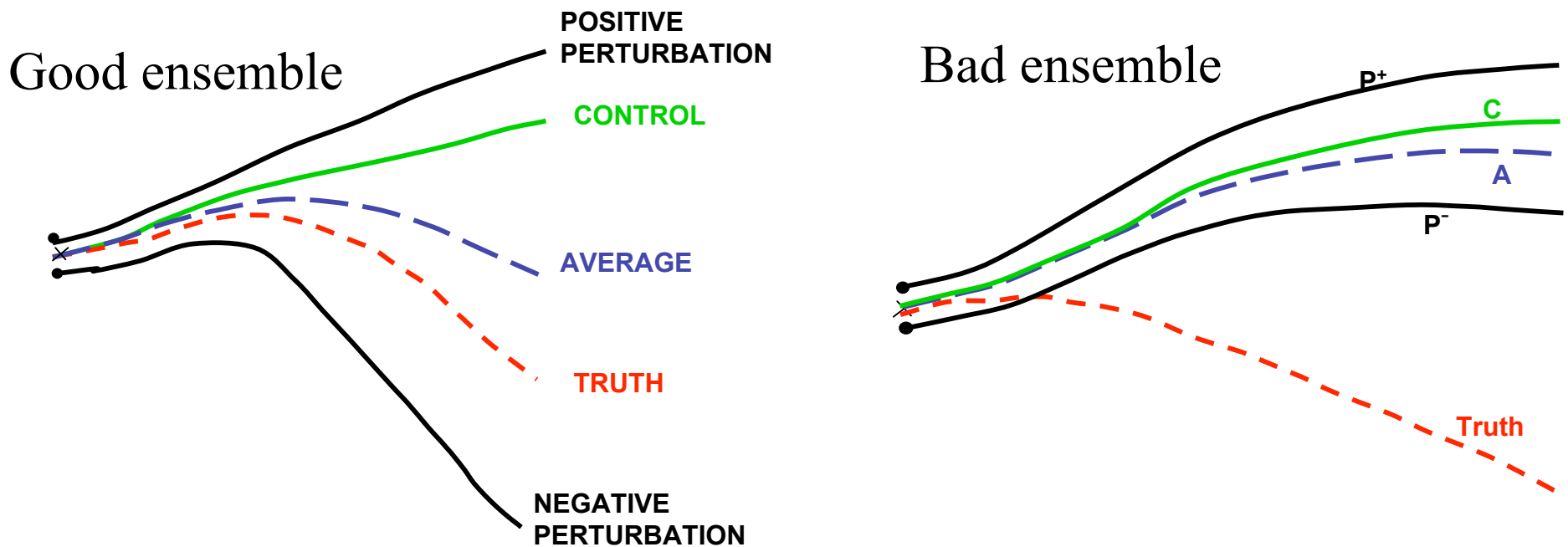
It makes sense to assume that the errors in the analysis (initial conditions) have the same shape as well: the errors lie in the subspace of the bred vectors

# Components of ensemble forecasts

An ensemble forecast starts from initial perturbations to the analysis...

In a good ensemble “truth” looks like a member of the ensemble

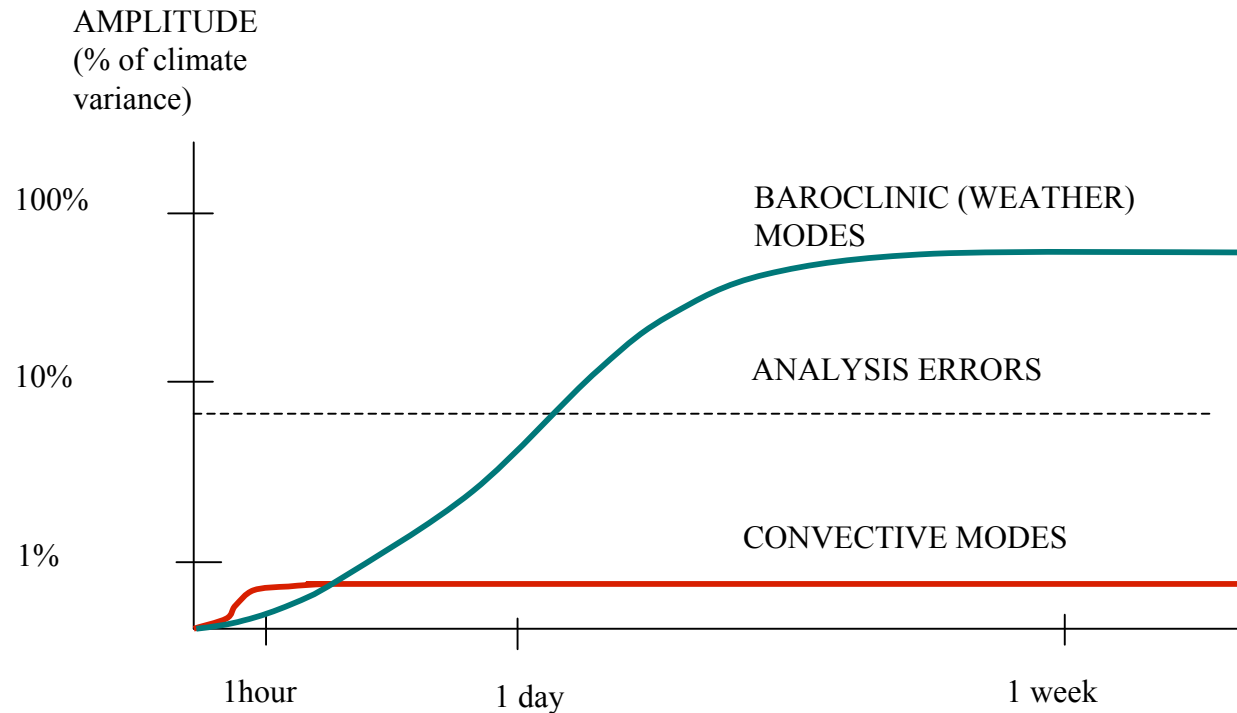
The initial perturbations should reflect the analysis “errors of the day”



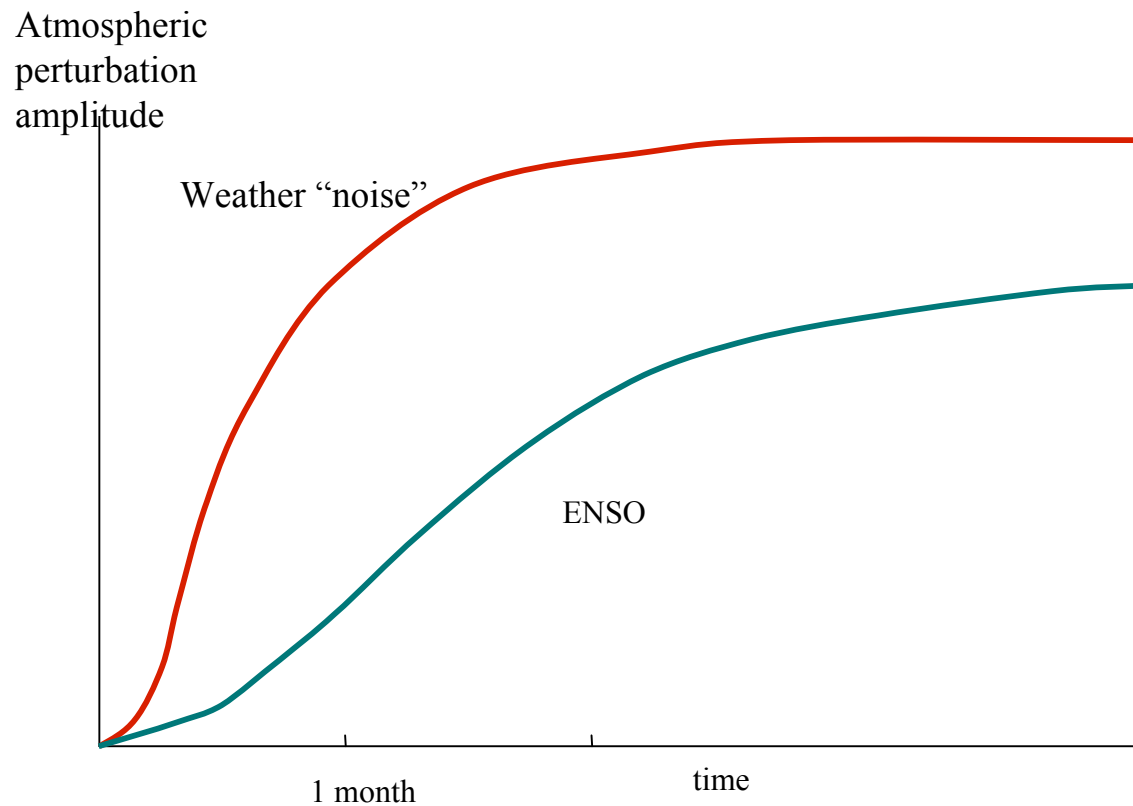
# Data assimilation and ensemble forecasting in a coupled ocean-atmosphere system

- A coupled ocean-atmosphere system contains growing instabilities with many different time scales
  - The problem is to isolate the slow, coupled instability related to the ENSO variability.
- Results from breeding in the Zebiak and Cane model (Cai et al., 2002) demonstrated that
  - The dominant bred mode is the slow growing instability associated with ENSO
  - The breeding method has potential impact on ENSO forecast skill, including postponing the error growth in the “spring barrier”.
- Results from breeding in a coupled Lorenz model show that using **amplitude** and **rescaling intervals** chosen based on time scales, breeding can be used to separate slow and fast solutions in a coupled system.

Nonlinear saturation allows filtering unwanted fast, small amplitude, growing instabilities like convection (Toth and Kalnay, 1993)



In the case of coupled ocean-atmosphere modes, we cannot take advantage of the small amplitude of the “weather noise”!  
We can only use the fact that the coupled ocean modes are slower...



We coupled a slow and a fast  
Lorenz (1963) 3-variable model

Fast equations

$$\frac{dx_1}{dt} = \sigma(y_1 - x_1) - C_1(Sx_2 + O)$$

$$\frac{dy_1}{dt} = rx_1 - y_1 - x_1z_1 + C_1(Sy_2 + O)$$

$$\frac{dz_1}{dt} = x_1y_1 - bz_1 + C_1(Sz_2)$$

Slow equations

$$\frac{1}{\tau} \frac{dx_2}{dt} = \sigma(y_2 - x_2) - C_2(x_1 + O)$$

$$\frac{1}{\tau} \frac{dy_2}{dt} = rx_2 - y_2 - Sx_2z_2 + C_2(y_1 + O)$$

$$\frac{1}{\tau} \frac{dz_2}{dt} = Sx_2y_2 - bz_2 + C_2(z_1)$$

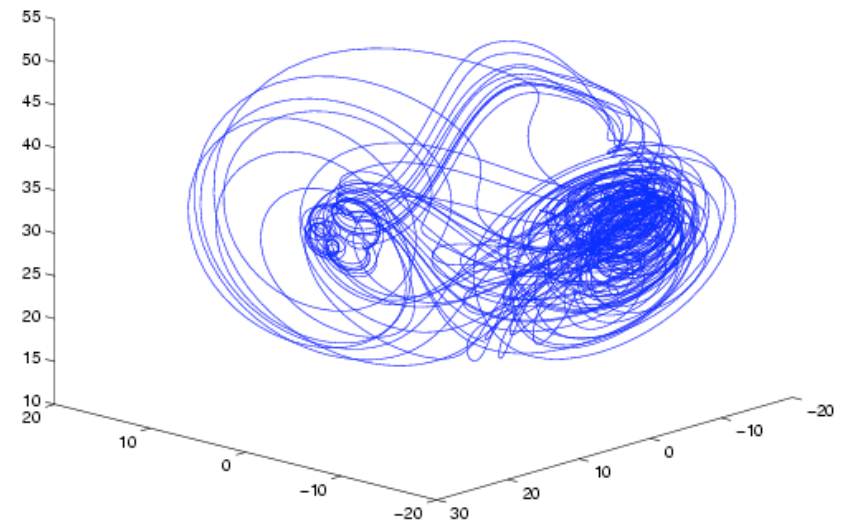
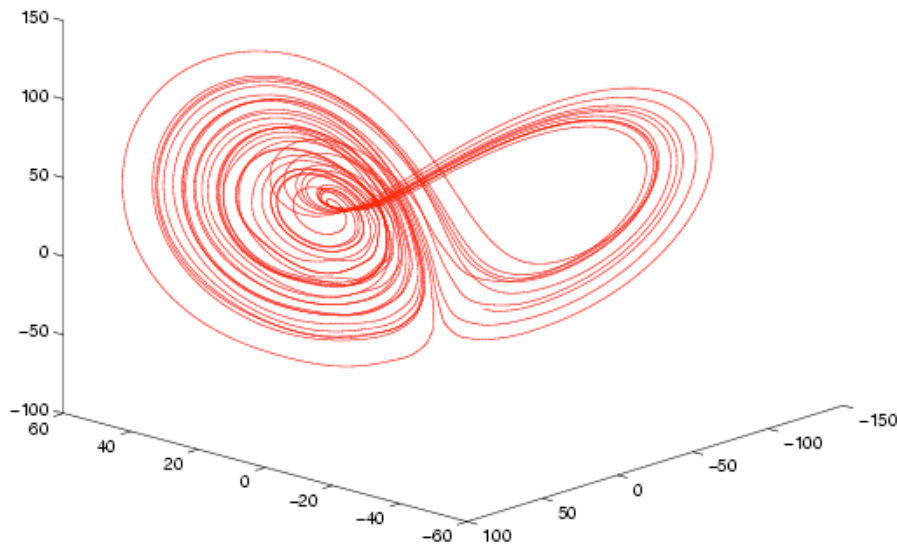
Now we test the fully coupled “ENSO-like” system,  
with similar amplitudes between “slow signal” and “fast noise”

“slow ocean”

“tropical atmosphere”

“Fully coupled Model (SLOW)”

“Fully coupled Model (FAST)”

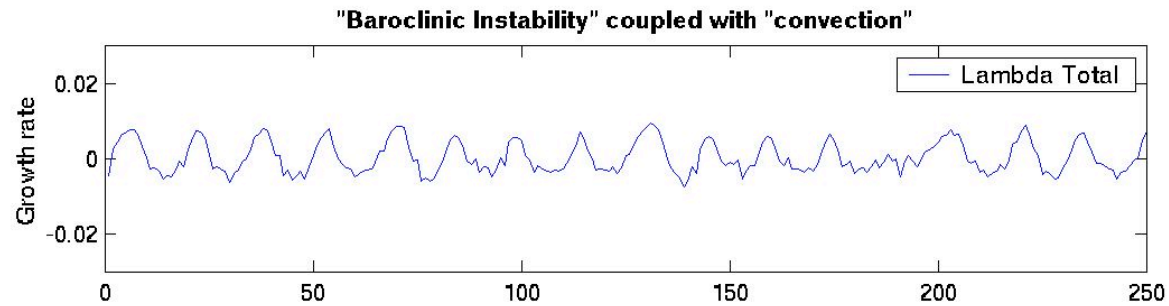


Then we added an extratropical atmosphere coupled with the tropics

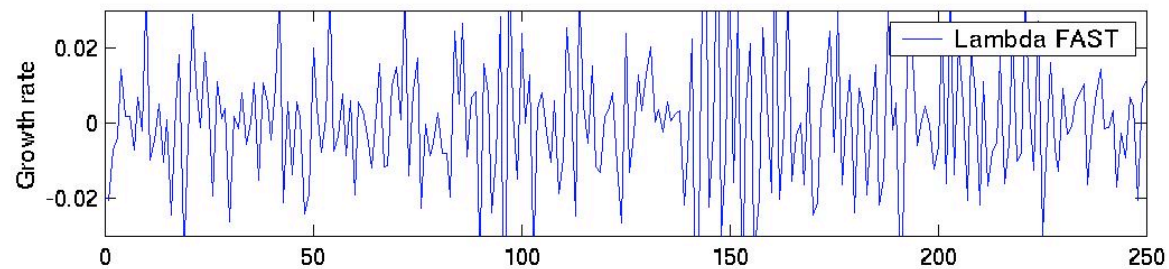


Depending on how we do the rescaling in the coupled model breeding, we can get the BVs for slow “weather waves” or fast “convection”

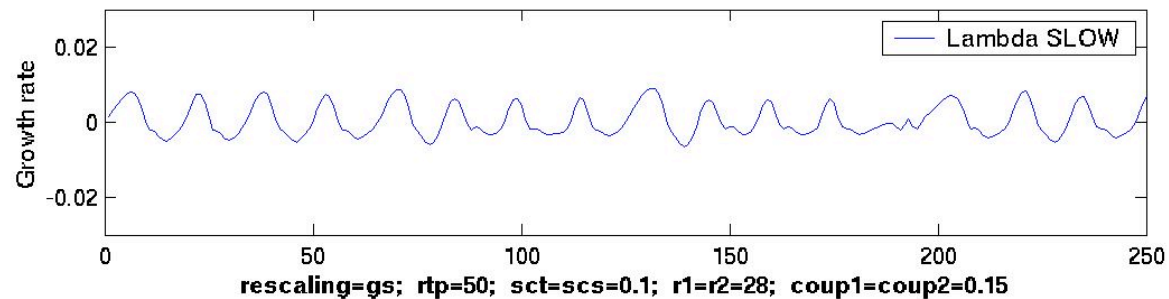
$\lambda_{total}$



$\lambda_{fast}$



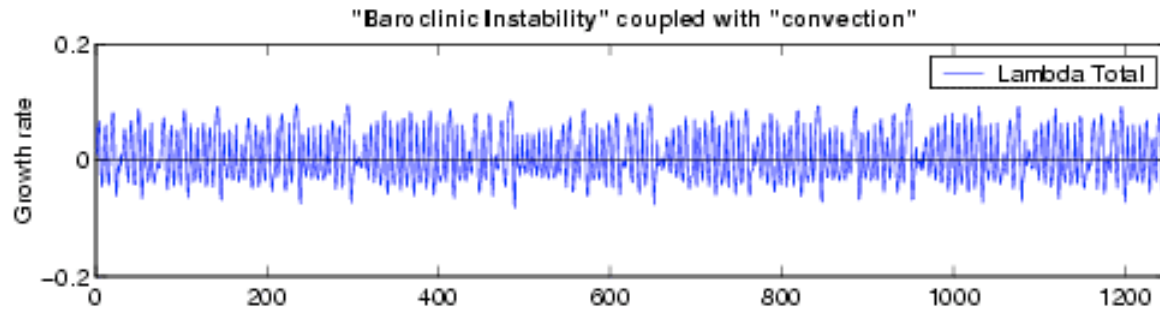
$\lambda_{slow}$



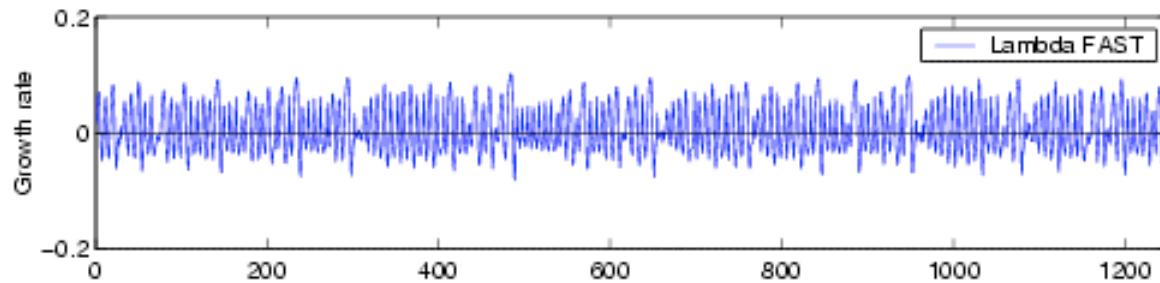
Rescaled with slow variables, slow frequency

If we **rescale with fast variable, at high frequency,**  
we get the “convection” bred vectors

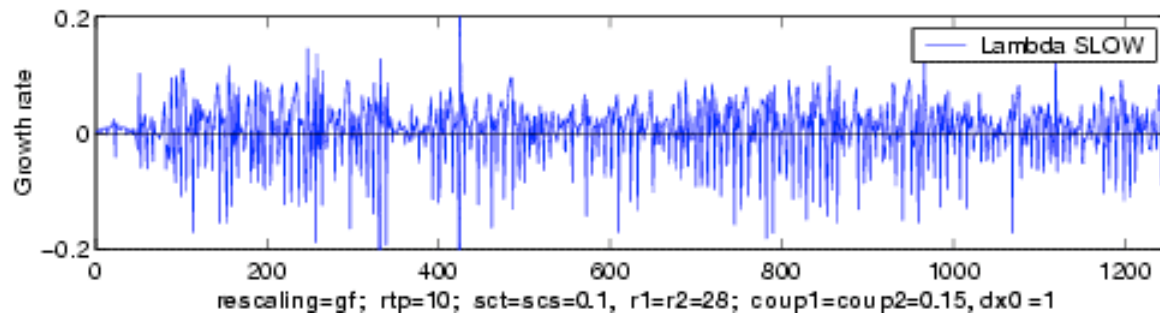
$\lambda_{total}$



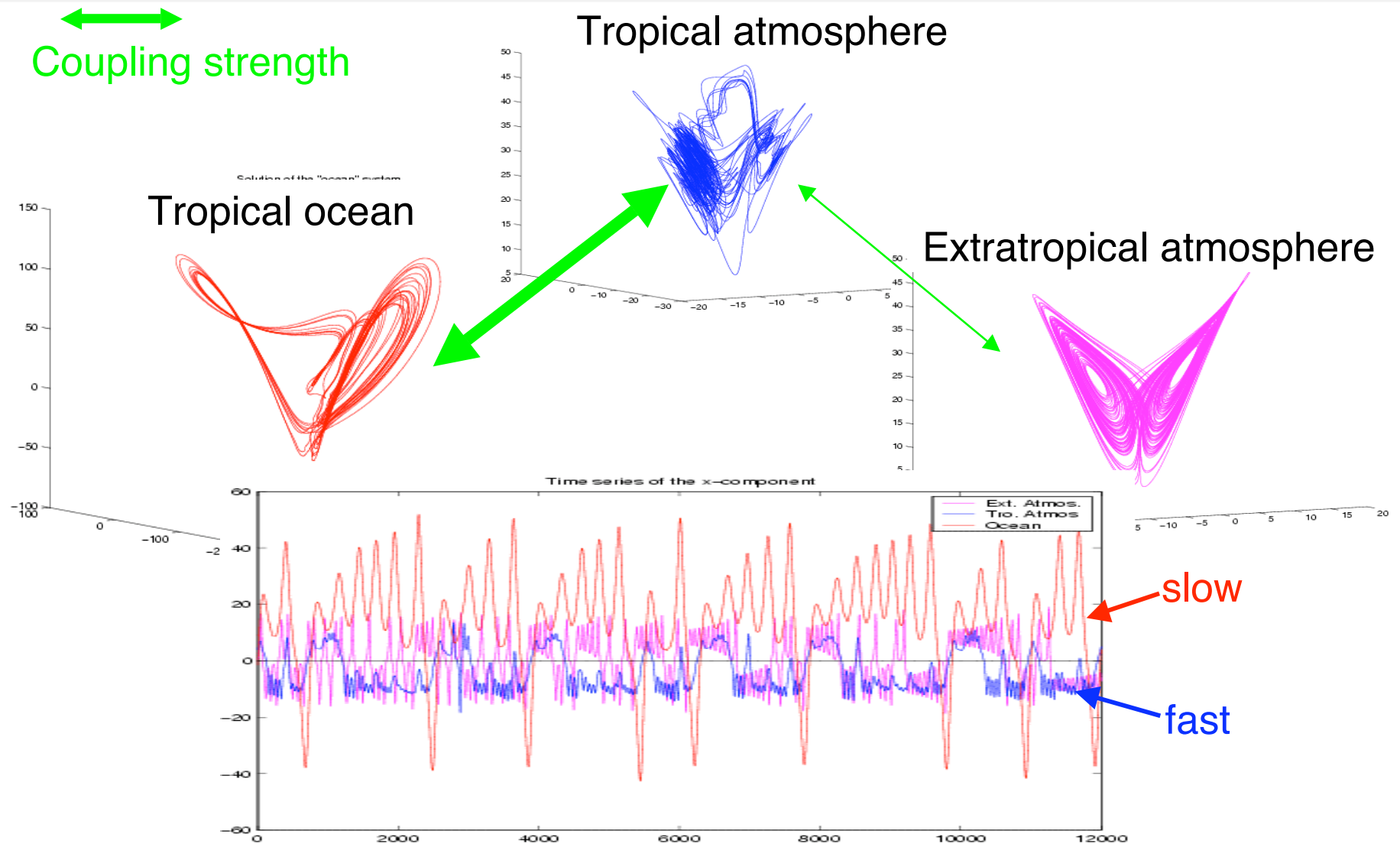
$\lambda_{fast}$



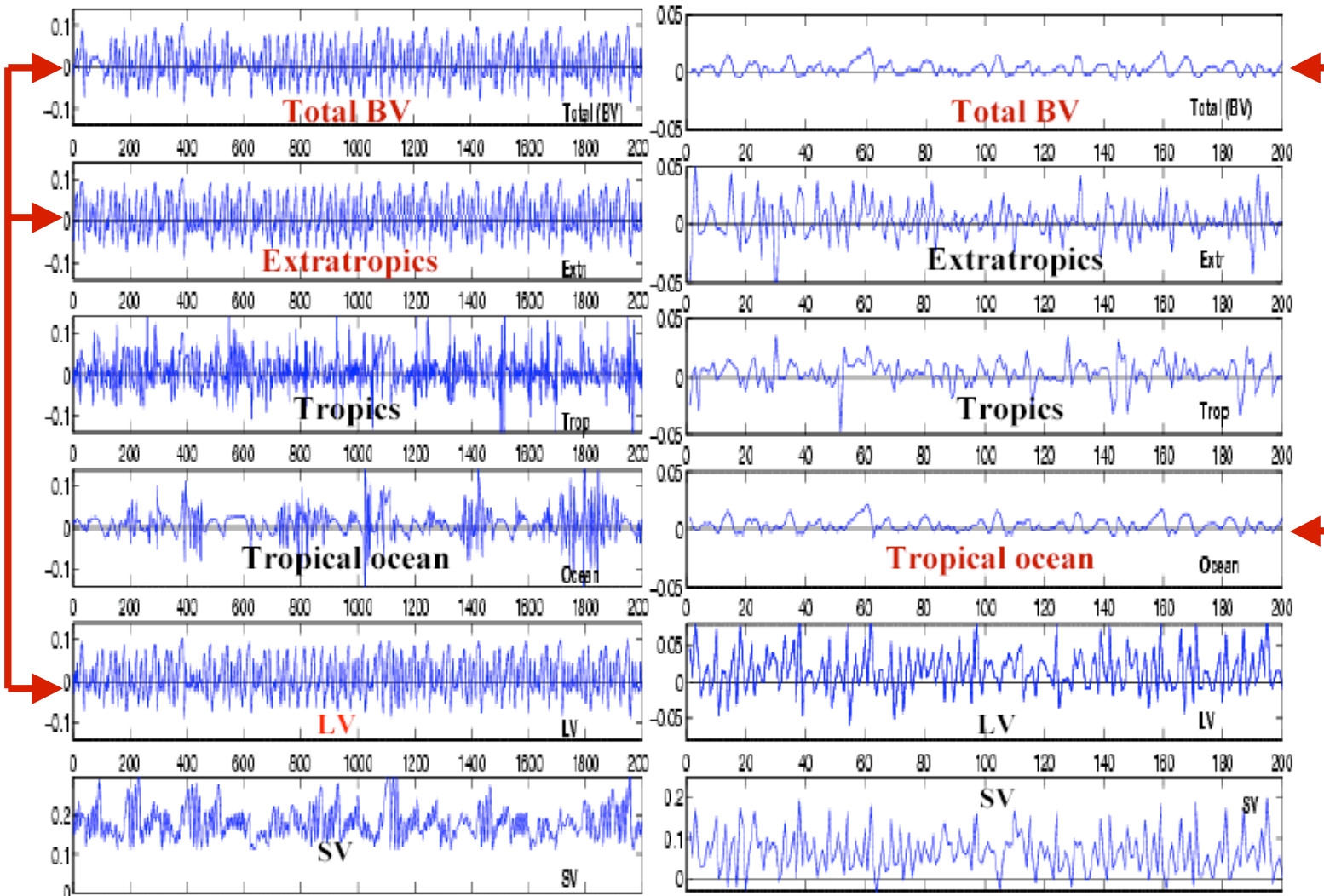
$\lambda_{slow}$



# Coupled fast and **slow** Lorenz 3-variable models (Peña and Kalnay, 2004)



# Breeding in a coupled Lorenz model



Short rescaling interval (5 steps)  
and small amplitude: fast modes

Long rescaling interval (50 steps)  
and large amplitude: ENSO modes

The linear approaches (LV, SV) cannot capture the slow ENSO signal

## From Lorenz coupled models:

- In coupled fast/slow models, we can do breeding to isolate the slow modes
- We have to choose **a slow variable** and **a long interval** for the rescaling
- This is true for nonlinear approaches (e.g., EnKF) but not for linear approaches (e.g., SVs, LVs)
- We apply this to ENSO coupled instabilities:
  - Cane-Zebiak model (Cai et al, 2003)
  - **NASA and NCEP fully coupled GCMs (Yang et al, 2006)**
  - **NASA operational system with real observations**