AOSC 614

Chapter 4

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4.1 Introduction

The role of parameterization in the numerical model

- Why do we need parameterization process?

Chapter 2 discussed the governing equations. Chapter 3 discussed the method to discretize these equations and computed numerically, however,

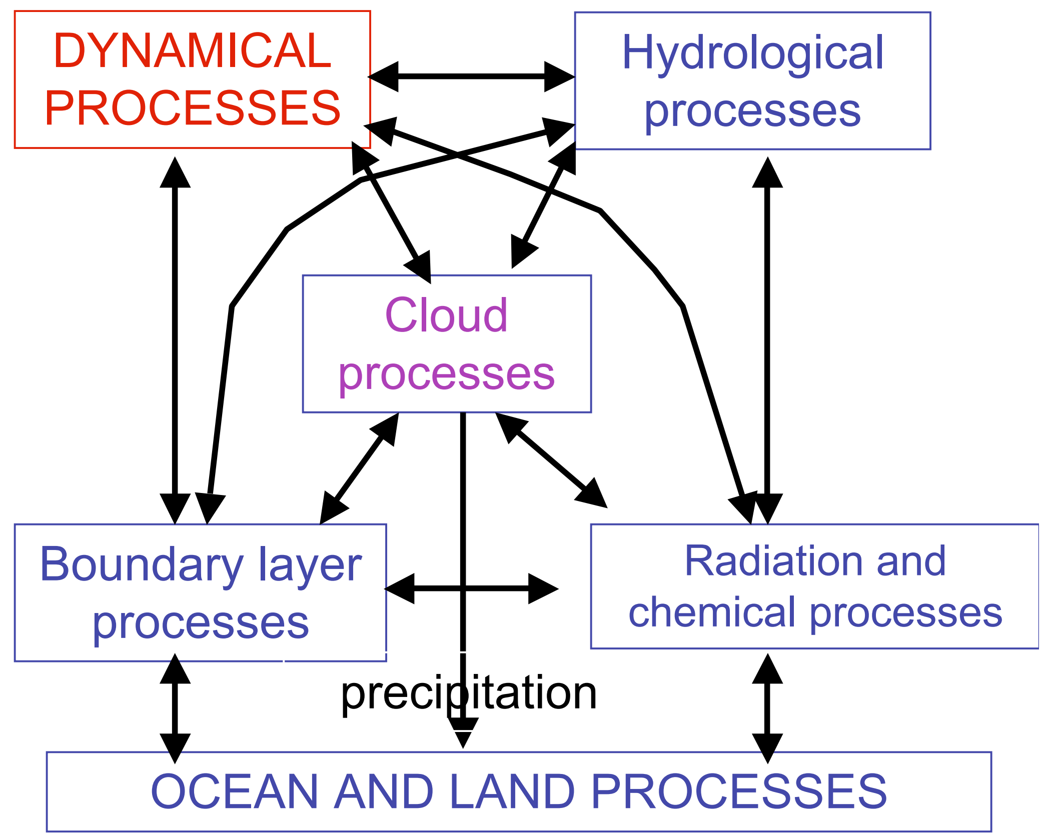
- ✓ The discretization of the continuous governing equation is limited by the model resolution, I.e., by the size of the smallest resolvable scale.
- ✓ In a finite difference scheme, the smallest scales of motion that can be resolved are those which have a wavelength of two grid sizes.
- ✓ Despite the continued increase of horizontal and vertical resolution, there are many processes and scales of motion that cannot be explicitly resolved with present of future models.

Subgrid-scale processes and interaction between subgrid-scale processes and resolved processes

✓ Subgrid-scale scale processes: all the processes that cannot be resolved explicitly by the model

✓ These subgrid-scale processes depend on and in turn, affects the large-scale fields and processes that are explicitly resolved by numerical models.

✓Cloud processes play a central role in the interaction between different processes



Dynamical processes are explicitly computed, while the other terms are subscale processes

Interaction between subgrid-scale processes and resolved processes

✓Cloud processes couple the dynamical and hydrological processes in the atmosphere

Through the heat of condensation and through the redistribution of sensible and latent heat, it changes the temperature field and the momentum (dynamical processes). Through condensation and evaporation, it changes the humidity field (dynamical processes)

✓Cloud processes couple radiative and dynamical-hydrological processes in the atmosphere through the reflection, absorption, and emission of radiation

✓ Cloud processes influence hydrological process in the ground through precipitation

✓ Cloud process influence the couplings between the atmosphere and oceans through modifications of radiation and planetary boundary layer (PBL) processes

Arakawa, 2004

Another interpretation of subgrid scale processes: adjustment processes

- Subgrid-scale processes can be interpreted as adjustment processes
- ✓ The atmospheric adjusts to the surface conditions through boundary layer adjustment processes.
- ✓ Convective processes occur in the presence of an unstable stratification and adjust towards a more neutrally stable state

The subgrid scale processes play important role in the atmospheric adjustment processes, and also their interactions with dynamical processes determine the important role they play in the numerical model.

4.2 Subgrid-scale processes and Reynolds averaging

Example: Prognostic equation for water vaper

Flux form in z-coordinates:

$$\frac{\partial \rho q}{\partial t} = -\frac{\partial \rho u q}{\partial x} - \frac{\partial \rho v q}{\partial y} - \frac{\partial \rho w q}{\partial z} + \rho E - \rho C \qquad (4.2.1)$$

Both u and q contain model grid scale and subgrid-scale processes , so

$$\begin{aligned} u &= \overline{u} + u' \\ q &= \overline{q} + q' \end{aligned} \qquad (4.2.2)$$

The overbar represents the spatial average over a grid, and the primes, the subgrid-scale perturbations

The Reynolds averaging rule:

$$\overline{q'} = 0, \overline{u'\bar{q}} = 0$$

$$\overline{\overline{uq}} = \overline{uq}$$

Substitute (4.2.2) into (4.2.1), and apply the Reynolds averaging rule:

$$\frac{\partial \rho \bar{q}}{\partial t} = -\frac{\partial \rho \bar{u}\bar{q}}{\partial x} - \frac{\partial \rho \bar{v}\bar{q}}{\partial y} - \frac{\partial \rho \bar{w}\bar{q}}{\partial z} - \frac{\partial \rho \overline{u'q'}}{\partial x} - \frac{\partial \rho \overline{v'q'}}{\partial y} - \frac{\partial \rho \overline{w'q'}}{\partial z} + \rho E - \rho C \quad (4.2.3)$$

- The first three terms of the right-hand side are the resolvable grid-scale advection (explicitly computed in the dynamical process)

- The next three terms are the turbulent moisture transports (not resolvable by dynamical equation, need to be parameterized)

- The last two terms are evaporation and condensation need to be parameterized. (not resolvable by dynamical equation)

Momentum equation and thermal dynamical equation

Applying the same rule to momentum and thermal dynamical equation:

$$\frac{\partial \bar{u}}{\partial t} = -\frac{\partial \overline{u}\overline{u}}{\partial x} - \frac{\partial \overline{u}\overline{v}}{\partial y} - \frac{\partial \overline{u}\overline{w}}{\partial z} - \frac{1}{\rho_o}\frac{\partial \bar{p}}{\partial x} + f\bar{v} - [\frac{\partial \overline{u'u'}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z}] + Friction$$

$$\frac{\partial \bar{v}}{\partial t} = -\frac{\partial \overline{u}\overline{v}}{\partial x} - \frac{\partial \overline{v}\overline{v}}{\partial y} - \frac{\partial \overline{v}\overline{w}}{\partial z} - \frac{1}{\rho_o}\frac{\partial \bar{p}}{\partial y} - f\bar{u} - [\frac{\partial \overline{u'v'}}{\partial x} + \frac{\partial \overline{v'v'}}{\partial y} + \frac{\partial \overline{v'w'}}{\partial z}] + Friction$$

$$\frac{\partial \bar{\theta}}{\partial t} = -\frac{\partial \bar{u}\theta}{\partial x} - \frac{\partial \theta\bar{v}}{\partial y} - \frac{\partial \theta\bar{w}}{\partial z} - \bar{w}\frac{d\theta_o}{dz} - [\frac{\partial \overline{u'\theta'}}{\partial x} + \frac{\partial \overline{\theta'v'}}{\partial y} + \frac{\partial \overline{\theta'w'}}{\partial z}] + Heating$$

$$\frac{\partial \rho\bar{q}}{\partial t} = -\frac{\partial \rho\overline{uq}}{\partial x} - \frac{\partial \rho\overline{vq}}{\partial y} - \frac{\partial \rho\overline{wq}}{\partial z} - \frac{\partial \rho\overline{u'q'}}{\partial x} - \frac{\partial \rho\overline{v'q'}}{\partial y} - \frac{\partial \rho\overline{w'q'}}{\partial z} + \rho E - \rho C$$

(4.2.3)

How to parameterize the unresolved term in the above equation?

Parameterization of the turbulent flux terms:

a. Bulk parameterization (slab model)

Assumption: the grid-scale field is well mixed in the boundary layer.

$$-\rho\overline{w'q'} = 0 \qquad (4.2.4)$$

In nature:

Convective boundary layer is topped by a stable layer

- Over land, during the day when surface heating is strong
- Over ocean, when the air near the sea surface is colder than the surface water temperature

b. K-theory

Assumption: Turbulent mixing acts in a manner analogous to molecular diffusion. The flux of a given field is proportional to the local gradient of the mean

$$-\rho\overline{w'q'} = K\frac{\partial \overline{q}}{\partial z} \qquad (4.2.5)$$

K is the eddy diffusivity.

In nature:

Neutrally or stably stratified boundary layer with moisture varying significantly with height.

Problem: how to determine the K since it depends on the stability basic flow and grid-average fields

C. Directly obtain a prognostic equation for the flux term

Vertical motion equation:

$$\frac{\partial w}{\partial t} = -u\frac{\partial w}{\partial x} - v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Multiply the above equation by ρq

$$\rho q\frac{\partial w}{\partial t} = -\rho q u\frac{\partial w}{\partial x} - \rho q v\frac{\partial w}{\partial y} - \rho q w\frac{\partial w}{\partial z} - \rho q\frac{1}{\rho}\frac{\partial p}{\partial z} - \rho q g \qquad \text{(a)}$$

(4. 2.1) multiplied by w:

$$w\frac{\partial \rho q}{\partial t} = -w\frac{\partial \rho u q}{\partial x} - w\frac{\partial \rho v q}{\partial y} - w\frac{\partial \rho w q}{\partial z} + w\rho E - w\rho C \qquad \text{(b)}$$

(a)+(b) then take Reynolds averaging, get prognostic equation:

$$\frac{\partial \rho\overline{w'q'}}{\partial t} = \cdots\frac{\partial \rho\overline{w'w'q'}}{\partial z}\cdots \qquad (4.2.7)$$

$$-\rho\overline{w'w'q'} = K'\frac{\partial \rho\overline{w'q'}}{\partial z}$$

Second order problem (Moeng and Wyngaard, 1989)

4.3 Model parameterizations

Atmospheric governing equatioin

$$\frac{d\overline{v}}{dt} = -\nabla_p \overline{\phi} - fk \times \overline{v} - g\frac{\partial \overline{\tau}}{\partial p} \qquad (4.3.1)$$

Momentum equation

$$\frac{\partial \overline{\phi}}{\partial p} = -\overline{\alpha} \qquad (4.3.2)$$

Hydrostatic equation

$$\nabla_p \cdot \overline{v} + \frac{\partial \overline{\omega}}{\partial p} = 0 \qquad (4.3.3)$$

Continuity equation

$$\frac{\partial \overline{p}_s}{\partial t} + \overline{v} \cdot \nabla \overline{p}_s = -\int_0^\infty \nabla_p \cdot \overline{v} dp \qquad (4.3.4)$$

Surface pressure

$$\overline{p}\overline{\alpha} = R\overline{T} \qquad (4.3.5)$$

Atmospheric state

$$C_p \frac{\overline{T}}{\overline{\theta}} \frac{d\overline{\theta}}{dt} = \tilde{Q} = \tilde{Q}_{rad} - g\frac{\partial \tilde{F}_\theta}{\partial p} + L(\tilde{C} - \tilde{E}) \qquad (4.3.6)$$

First law of thermodynamics

$$\frac{d\overline{q}}{dt} = \tilde{E} - \tilde{C} - g\frac{\partial \tilde{F}_q}{\partial p} \qquad (4.3.7)$$

Water vapor conservation eq.

Terms in this equation

- The overbar are the grid-averaged quantities computed by the model dynamics, and the terms with the tilde represent subscale processes that are need to be parameterized.

✓ Momentum equation has the effect of eddy fluxes of momentum

✓ Thermal dynamics equation includes radiative heating and cooling, sensible heat fluxes and condensation and evaporation

✓ The water vapor equation includes the condensation and evaporation, and the moisture flux.

Type of parameterization processes:

➢ Vertical flux terms $\frac{\partial \tilde{\tau}}{\partial p}, \frac{\partial \tilde{F}_\theta}{\partial p}, \frac{\partial \tilde{F}_q}{\partial p}$

➢ Radiative heating and cooling $\tilde{Q}_{rad}$

➢ Condensation and evaporation $\tilde{E} - \tilde{C}$

Vertical flux term

Eddy flux of momentum, of sensible heat, and of moisture is written as (neglecting horizontal turbulence because of scales):

$$\tilde{\tau} = \rho \overline{w'u'}\mathbf{i} + \rho \overline{w'\upsilon'}\mathbf{j}$$

$$\tilde{F}_\theta = \rho C_p \overline{w'\theta'}$$

$$\tilde{F}_q = \rho \overline{w'q'}$$

These terms can be represented using K-theory in the boundary layer and neglected in the free atmosphere above boundary by setting K=0

Derivation of surface flux terms

- The vertical derivative of the lower boundary turbulent fluxes require surface fluxes of heat, moisture, and momentum.
- One of the most used surface flux parameterization is the bulk parameterization based on the Monin-Obukhov (1954) similarity theory

$$\tau = -\rho C_D |\mathbf{v}| \mathbf{v}$$

$$F_\theta = -\rho C_H |\mathbf{v}| C_p (\theta - \theta_S)$$

$$F_q = -\rho C_E |\mathbf{v}| \beta (q - q_S)$$

➢The wind and temperature in the surface layer can be described by a set of equations that depends only on a few parameters (e.g, roughness length)

➢ The theory suggests that the flux is constant with height in the surface layer

$\mathbf{v}, \theta, q$ Are the velocity, potential temperature, and mixing ratio in the surface layer, respectively, and the variables with an S subscript are the corresponding values at the underlying ocean or land surface $(\mathbf{v}_S = 0)$

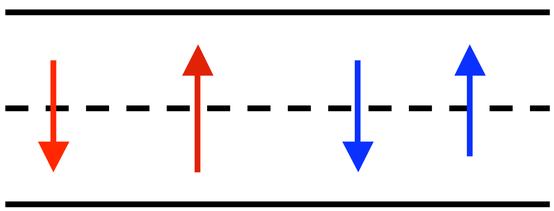
$C_D, C_H and C_E$ are transfer coefficients and they depend on the stability of the surface layer.

Radiation

- It is determined from the vertical divergence of the upward and downward fluxes of short- and long- wave radiation, obtained using the radiative transfer equation. (Kiehl, 1992)

$$\tilde{Q}_{rad}^{LR} = {}^{d}F^{LR} - {}^{u}F^{LR}$$

$$\tilde{Q}_{rad}^{SR} = {}^{d}F^{SR} - {}^{u}F^{SR}$$



The interactions between clouds and radiation are important. How to determine clouds?

- Cloud from climatology (Manabe et al., 1965)
- Cloud cover is based on relative humidity (Slingo 1987)
- Cloud and rain water were predicted using budget equations and cloud cover was derived from the amount of cloud water (Zhao et al., 1997)

Cloud properties are also important

- Cloud has plane slab structure? Not really…
- Clouds have a fractal structure?

4.4 Parameterization in SPEEDY model (simple)

Large-scale condensation

When the large scale predicts supersaturation the model forms clouds:

- Relative humidity $RH > RH(\sigma_k)_{lsc}$, the specific humidity is relaxed towards the corresponding threshold value on the time-scale of 4 hours

- With the condensation of the water vapor, the latent heat content is used to change the temperature tendency, and converted into dry static energy

Specific humidity tendency:

$$(\frac{\partial Q_k}{\partial t})_{lsc} = -\frac{Q_k - RH(\sigma_k)_{lsc} Q_k^{sat}}{\tau_{lsc}}$$

Temperature tendency:

$$(\frac{\partial T_k}{\partial t})_{lsc} = -\frac{L_c}{c_p}(\frac{\partial Q_k}{\partial t})_{lsc}$$

The precipitation flux

$$P_{lsc} = -\frac{1}{g}\sum_{k=2}^{N} \Delta p_k (\frac{\partial Q_k}{\partial t})_{lsc}$$

Dry static energy

$$SE = c_p T + \phi$$

Convection

Simplified version of mass flux scheme developed by Tiedke (1993)

Condition:

a. conditional instability is present

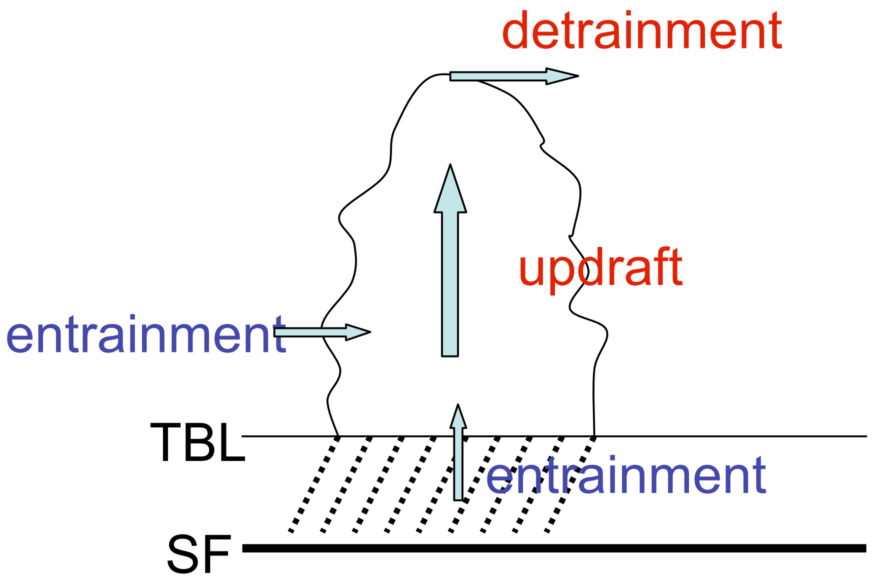
b. The relative humidity in the PBL exceeds a prescribed threshold

The process:

• The entrainment is such that the PBL humidity is relaxed towards the threshold value on a time-scale of 6 hours

• Detrainment occurs only at the cloud top level

• The air in the updraft is saturated, no evaporation



The updraft only changes the moisture tendency term, the temperature tendency term is changed in the detrainment level

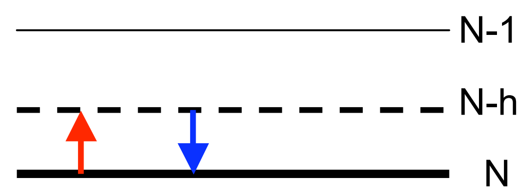
Convection

Simplified version of mass flux scheme developed by Tiedke (1993)

Conditions for convection:

- ✓ conditional instability is present:
  Saturation moist static energy (MSS=SE + LcQ^{sat}) decreases with height
- ✓ Humidity exceeds a prescribed threshold in both the PBL and one level above

With the grid points that satisfy the above equation, first define the fluxes of mass (m), humidity and dry static energy at the top of the PBL



$$ {}^{u}F^{m}_{N-h} = {}^{d}F^{m}_{N-h} = F^{*} $$

$$ {}^{u}F^{Q}_{N-h} = F^{*} \cdot Q^{sat}_{N} ; \; {}^{d}F^{Q}_{N-h} = F^{*} \cdot Q_{N-h} $$

$$ {}^{u}F^{SE}_{N-h} = F^{*} \cdot SE_{N} ; \; {}^{d}F^{SE}_{N-h} = F^{*} \cdot SE_{N-h} $$

Substitute back

$$ F^{*} = \frac{\Delta p_N}{g\tau_{cnv}} \frac{Q_N - Q_{thr}}{Q^{sat}_N - Q_{N-h}} $$

$$ \left(\frac{\partial Q_N}{\partial t}\right)_{cnv} = -\frac{gF^{*}(Q^{sat}_N - Q_{N-h})}{\Delta p_N} = -\frac{Q_N - Q_{thr}}{\tau_{cnv}} $$

PBL

Convection

In the 'intermediate' layers between PBL and TCN

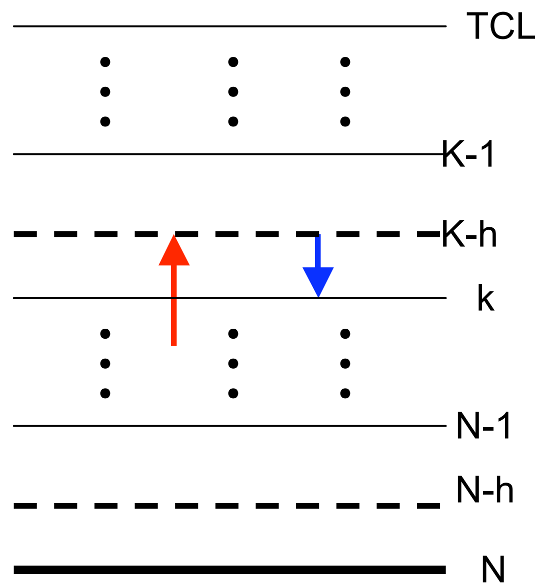
The upward fluxes are increased by entrainment (E^m)

$$E_k^m = \varepsilon(\sigma_k)F_{k+h}^m$$

$$F_{k-h}^m = F_{k+h}^m + E_k^m$$

$$^uF_{k-h}^Q = {}^uF_{k+h}^Q + E_k^m Q_k ;\ {}^dF_{k-h}^Q = F_{k-h}^m Q_{k-h}$$

$$^uF_{k-h}^{SE} = {}^uF_{k+h}^{SE} + E_k^m SE_k ;\ {}^dF_{k-h}^{SE} = F_{k-h}^m SE_{k-h}$$



Note: TCL (top of convection)

Assumption:

✓ No detrainment occurs in 'intermediate' layers, so no need to calculate condensation.

✓ All the energy is transported upward, and does not modify the local temperature tendency, so condensation only computes at TCL

Convection

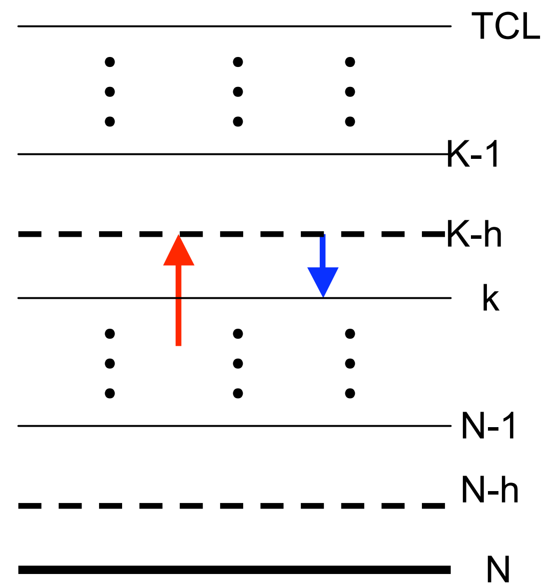
In TCL (top of convection layer)

$$P_{cnv} = {}^{u}F^{Q}_{k0+h} - F^{m}_{k0+h}Q^{sat}_{k0+h}(k0 = k(TCN))$$

The net flux of moisture and energy into TLC layer

$$\Delta F^{Q}_{k0} = {}^{u}F^{Q}_{k0+h} - {}^{d}F^{Q}_{k0+h} - P_{cnv}$$

$$\Delta F^{SE}_{k0} = {}^{u}F^{SE}_{k0+h} - {}^{d}F^{SE}_{k0+h} + L_c P_{cnv}$$



Therefore, change the moisture and temperature tendency term

Clouds

Theory:

- The cloud cover and thickness are defined diagnostically from the values of relative humidity in an air column including all tropospheric layers except the PBL, and the amount of total precipitation.

The existence of clouds:

Cloud base is at the interface between the lowest two model layers, and the top at the upper boundary of highest layer in which the following conditions are satisfied: $RH_k > RH_{cl}$, $Q_k > Q_{cl}$

The amount of clouds:

The cloud cover is determined by the sum of terms proportional to the square root of total precipitation and the square of the RHk-RHcl

Radiation

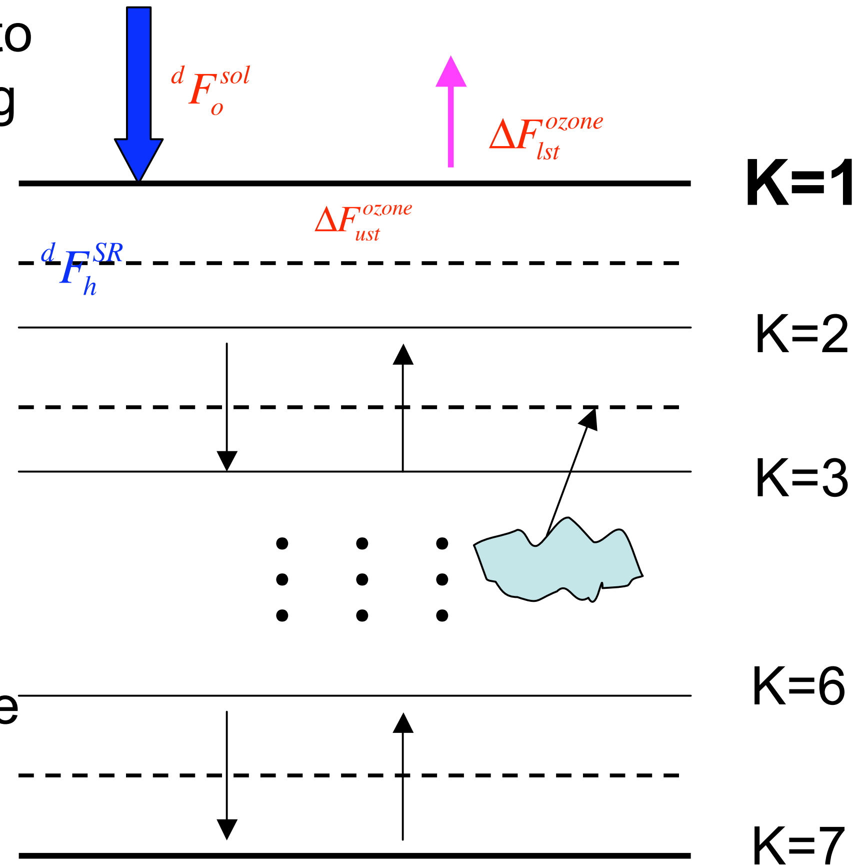
Shortwave radiation

• Consider two bands, one corresponding to visible component, the other corresponding to infrared.

• Radiation is reflected at cloud top and at the surface;

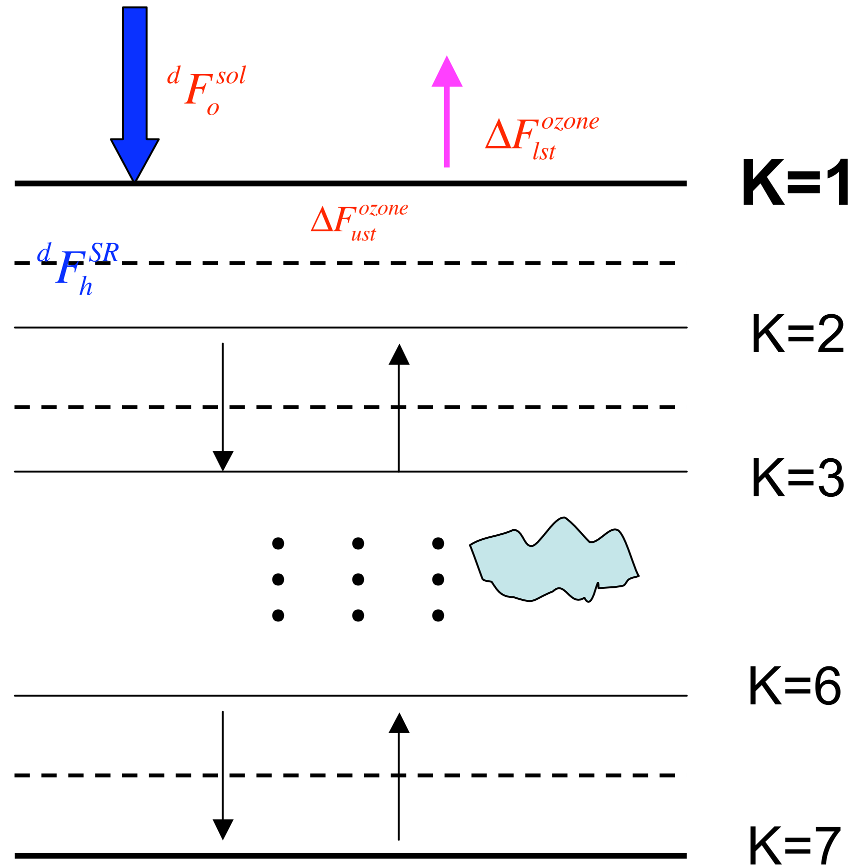
• The cloud albedo is proportional to the total cloud cover;

• The shortwave transmissivities of the the model layers are functions of layer mass, specific humidity and could cover. They are computed for each band and each layer



Radiation

Shortwave radiation



Downward SR in the first layer:

$$^{d}F_{h}^{SR} = {}^{d}F_{o}^{sol} - \Delta F_{ust}^{ozone} - \Delta F_{lst}^{ozone}$$

Downward SR in an intermediate layer with clouds:

$$^{d}F_{k+h}^{SR} = {}^{d}F_{k-h}^{SR}(1 - A_{cl})\tau_{k}^{SR}$$

Downward SR in the intermediate layer with no clouds:

$$^{d}F_{k+h}^{SR} = {}^{d}F_{k-h}^{SR}\tau_{k}^{SR}$$

Upward of SR in the surface

$$^{u}F_{s}^{SR} = {}^{d}F_{s}^{SR}A_{s}$$

Upward SR in the intermediate layer

$$^{u}F_{k-h}^{SR} = {}^{u}F_{k+h}^{SR}\tau_{k}^{SR}$$

The net radiative forcing of SR:

$$\tilde{Q}_{rad}^{SR} = {}^{d}F_{k+h}^{SR} - {}^{u}F_{k+h}^{SR}$$

forcing for the thermal-dynamics equation

$$C_p \frac{\bar{T}}{\bar{\theta}} \frac{d\bar{\theta}}{dt} = \tilde{Q} = \tilde{Q}_{rad} - g\frac{\partial \tilde{F}_{\theta}}{\partial p} + L(\tilde{C} - \tilde{E})$$

Longwave radiation

The infrared spectrum is partitioned into four regions

a. The “infrared window” between 8.5 and 11μm (band1)
b. The band of strong absorption by CO2 around 15 μm (band 2)
c. The weak or moderate absorption by water vapor (band 3)
d. The regions with strong absorption by water vapor (band 4)

Longwave radiation

Transmissivity

A transmissivity is computed for each band and model layer as a function of layer depth, humidity and cloud properties.

Transmissivity in the window band with :

$$\tau_{k,1}^{LR} = \exp[-(\alpha_{win}^{LR} + \alpha_{cl}^{LR} CLC)\Delta p_k']$$ In cloudy region

$$\tau_{k,1}^{LR} = \exp(-\alpha_{win}^{LR} \Delta p_k')$$ No cloud

α Coefficients are constant absorptivity parameters

Transmissivity in the water and CO2 bands:

$$\tau_{k,2}^{LR} = \exp(-\alpha_{co_2}^{LR} \Delta p_k')$$

$$\tau_{k,3}^{LR} = \exp(-\alpha_{wv1}^{LR} q_k \Delta p_k')$$

$$\tau_{k,3}^{LR} = \exp(-\alpha_{wv2}^{LR} q_k \Delta p_k')$$

Longwave radiation

Downward and upward longwave radiation

Precondition: $^{d}F_{k=1}^{LR} = 0$

Downward longwave radiation:

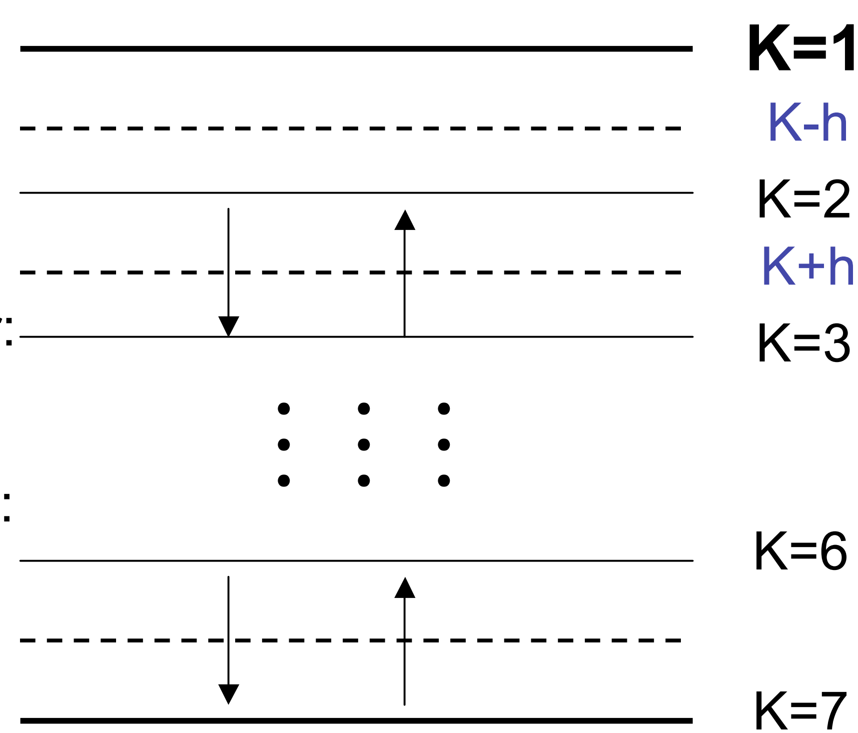
$$^{d}F_{k+h}^{LR} = {}^{d}F_{k-h}^{LR}\tau_{k}^{LR} + (1-\tau_{k}^{LR})\,{}^{d}B_{k}$$

Upward longwave radiation in each layer:

$$^{u}F_{k-h}^{LR} = {}^{u}F_{k+h}^{LR}\tau_{k}^{LR} + (1-\tau_{k}^{LR})\,{}^{u}B_{k}$$

Upward longwave radiation from surface:

$$^{u}F_{s}^{LR} = \varepsilon_{s} f_{b}(T_{s})\,{}^{u}B_{s} + (1-\varepsilon_{s})\,{}^{d}F_{s}^{LR}$$



$$\tilde{Q}_{rad}^{LR} = {}^{d}F_{k+h}^{LR} - {}^{u}F_{k+h}^{LR}$$

We get part of the forcing for the thermal-dynamics equation

$$C_{p}\frac{\bar{T}}{\bar{\theta}}\frac{d\bar{\theta}}{dt} = \tilde{Q} = \tilde{Q}_{rad} - g\frac{\partial \tilde{F}_{\theta}}{\partial p} + L(\tilde{C} - \tilde{E})$$

Surface fluxes

❖ Theory: In SPEEDY, surface fluxes are modelled using aerodynamic formulas. Since the PBL is represented by just one layer, one cannot use variables at the lowest model level as approximations of near-surface variables. Also, vertical gradients between two model levels cannot be used to estimate PBL stability properties.

STEPS

✓ Estimation of near-surface atmospheric values of wind, temperature and humidity.

➢ Wind is assumed to be proportional to the wind at the lowest full level:

$$U_{sa} = f_{wind} U_N ; V_{sa} = f_{wind} V_N$$

➢ For temperature and moisture, two options are available:

A: near-surface values of potential temperature and specific humidity are the same as the lowest model level

B: Temperature is extrapolated at $\sigma = 1$ using values at the two lowest levels(T_N , T_{N-1}) and assuming a linear profile in $\log(\sigma)$

Humidity: $RH_{sa} = RH_N ; q_{sa} = q_{sa}^{sat}(T_{sa}) \times RH_{sa}$

Surface fluxes

Surface stresses, sensible heat flux and evaporation are computed over land surface as:

$$V_0 = (U_{sa}^2 + V_{sa}^2 + V_{gust}^2)^{1/2}$$

$$^{u}F_{ls}^{U} = \tau_{ls}^{U} = -\rho_{sa} C_l^D \left|V_0\right| U_{sa}$$

$$^{u}F_{ls}^{V} = \tau_{ls}^{V} = -\rho_{sa} C_l^D \left|V_0\right| V_{sa}$$

$$^{u}F_{ls}^{SE} = SHF_{ls} = \rho_{sa} C_l^H \left|V_0\right| c_p (T_{skin} - T_{sa})$$

$$^{u}F_{ls}^{Q} = E_{ls} = \rho_{sa} C_l^H \left|V_0\right| \max[\alpha_{sw} Q^{sat}(T_{skin}, p_s) - Q_{sa}, 0]$$

α_{sw} is soil-water availability index. It depends on vegetation fraction, and on soil moisture values at filed capacity and wilting point.

Over sea surface, surface stresses have the same formulation as over land, sensible heat and moisture fluxes:

$$^{u}F_{ss}^{SE} = SHF_{ss} = \rho_{sa} C_s^H \left|V_0\right| c_p (T_{sea} - T_{sa})$$

$$^{u}F_{ss}^{Q} = E_{ss} = \rho_{sa} C_s^H \left|V_0\right| \max[Q^{sat}(T_{sea}, p_s) - Q_{sa}, 0]$$

SUMMARY

Atmospheric governing equations

$$\frac{d\bar{v}}{dt} = -\nabla_p \bar{\phi} - fk \times \bar{v} - g\frac{\partial \bar{\tau}}{\partial p} \qquad (4.3.1)$$

$$\frac{\partial \bar{\phi}}{\partial p} = -\bar{\alpha} \qquad (4.3.2)$$

$$\nabla_p \cdot \bar{v} + \frac{\partial \bar{\omega}}{\partial p} = 0 \qquad (4.3.3)$$

$$\frac{\partial \bar{p}_s}{\partial t} + \bar{v} \cdot \nabla \bar{p}_s = -\int_0^\infty \nabla_p \cdot \bar{v} dp \qquad (4.3.4)$$

$$\bar{p}\bar{\alpha} = R\bar{T} \qquad (4.3.5)$$

$$C_p \frac{\bar{T}}{\bar{\theta}} \frac{d\bar{\theta}}{dt} = \tilde{Q} = \tilde{Q}_{rad} - g\frac{\partial \tilde{F}_\theta}{\partial p} + L(\tilde{C} - \tilde{E}) \qquad (4.3.6)$$

$$\frac{d\bar{q}}{dt} = \tilde{E} - \tilde{C} - g\frac{\partial \tilde{F}_q}{\partial p} \qquad (4.3.7)$$

✓The bars represent the grid-average variables, computed by the model dynamics.

✓The colored terms are the paramerizations of subgrid-scale processes

✓Once we have reasonable parameterization of subgrid-scale processes, we are ready to have numerical model based on the these equations.